Mathematics

Calculation Policy

Date policy adopted by The Bridge Federation:

Review date:

Contents

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Overview

Introduction and rationale:

This policy has been written in line with the programmes of study taken from the revised National Curriculum for Mathematics (2014), in which there is a greater emphasis on arithmetic and formal written methods of calculation.

The revised curriculum is also a 'mastery' curriculum, where mastery is a process or approach to learning for everyone, ensuring that all children secure the key concepts and skills to allow them to explore things in greater depth. The aim is to develop fluency and the ability to reason mathematically and solve problems; it is expected that there is a narrower spread of attainment, where the more able should be catered for through deepening their reasoning and problem solving skills rather than being accelerated into new curriculum content.

Each year group should spend longer time on key topics to develop depth before moving on. This will mean that there is less opportunity to revisit topics and so making connections to topics taught previously will be important to consolidate learning and develop fluency. When planning, significant time should be devoted to developing key number concepts in each year group, ensuring students build fluency.

This policy provides guidance on progression in calculation (part 1). A document for each operation addresses what and how to teach year by year (part 2), each of which lays out expectations for both mental and written calculations (generally collated for Key Stage 1), including calculation of fractions, and includes statutory statements from the national curriculum (in bold) together with the non-statutory guidance (in italics) and supplementary guidance (in plain text).

Key to successful implementation of calculation procedures is consistent use of representations (models and images that support conceptual understanding of the mathematics) and this policy promotes range of relevant representations (not exhaustive) and a side-by-side approach to teaching and learning (parts 2 and 3). Mathematical understanding is developed through use of representations that are first of all concrete (e.g. Numicon, Dienes apparatus), and then pictorial (e.g. array, place value counters) to then facilitate abstract working (e.g. columnar addition/ subtraction, short/long multiplication). This policy offers guidance through an appropriate progression of representations, and if at any point a pupil is struggling they should revert to familiar pictorial and/or concrete materials/ representations as appropriate. Whilst a mathematically fluent child will be able to choose the most appropriate representation and procedure to carry out a calculation, whether written or mental, schools should support pupils with carefully selected representations that underpin calculation methods and ensure there is consistency across year groups.

Aims:

- to ensure consistency and progression in our approach to calculation
- to ensure that children develop an efficient, reliable, formal written method of calculation for all operations
- to ensure that children can use these methods accurately with confidence and understanding

How to Use:

- use the policy as the basis of your planning but ensure you use previous years' guidance to allow for personalised learning
- always use Assessment for Learning to identify suitable next steps in calculation for groups of children
- if, at any time, children are making significant errors, return to the previous stage in calculation
- always use suitable resources, models and images to support children's understanding of calculation and place value, as appropriate
- encourage children to make sensible choices about the methods they use when solving problem

Calculation policy: Progression

	EYFS/Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on- using cubes. Regrouping to make 10 using ten frame.	Adding three single digits. Use of base 10 to combine two numbers.	Column method- regrouping. Using place value counters (up to 3 digits).	Column method- regrouping. (up to 4 digits)	Column method- regrouping. Use of place value counters for adding decimals.	Column method- regrouping. Abstract methods. Place value counters to be used for adding decimal numbers.
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10 using the ten frame	Counting back Find the difference Part whole model Make 10 Use of base 10	Column method with regrouping. (up to 3 digits using place value counters)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. Abstract for whole numbers. Start with place value counters for decimals- with the same amount of decimal places.	Column method with regrouping. Abstract methods. Place value counters for decimals- with different amounts of decimal places.

Multiplication	Recognising and making equal groups. Doubling Counting in multiples Use cubes, Numicon and other objects in the classroom	Arrays- showing commutative multiplication	Arrays 2d × 1d using base 10	Column multiplication- introduced with place value counters. (2 and 3 digit multiplied by 1 digit)	Column multiplication Abstract only but might need a repeat of year 4 first(up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication Abstract methods (multi-digit up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups? Use cubes and draw round 3 cubes at a time.	Division as grouping Division within arrays- linking to multiplication Repeated subtraction	Division with a remainder-using lollipop sticks, times tables facts and repeated subtraction. 2d divided by 1d using base 10 or place value counters	Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number including remainders)	Short division Long division with place value counters (up to 4 digits by a 2 digit number) Children should exchange into the tenths and hundredths column too

Number: Addition and Subtraction

National Centre for Excellence in the Teaching of Mathematics

NUMBER BONDS					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
represent and use number bonds and related subtraction facts within 20	recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100				
MENTAL CALCULATION					
add and subtract one-digit and two-digit numbers to 20, including zero	add and subtract numbers using concrete objects, pictorial representations, and mentally, including: * a two-digit number and ones * a two-digit number and tens * two two-digit numbers * adding three one-digit numbers	add and subtract numbers mentally, including: * a three-digit number and ones * a three-digit number and tens * a three-digit number and hundreds		add and subtract numbers mentally with increasingly large numbers	perform mental calculations, including with mixed operations and large numbers
read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs (appears also in Written Methods)	show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot				use their knowledge of the order of operations to carry out calculations involving the four operations

Number: Addition and Subtraction

National Centre for Excellence in the Teaching of Mathematics

WRITTEN METHODS					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs (appears also in Mental Calculation)		add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction	add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate	add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)	
INVERSE OPERATIONS, ESTIMATING AND CHECKING ANSWERS					
	recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.	estimate the answer to a calculation and use inverse operations to check answers	estimate and use inverse operations to check answers to a calculation	use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy	use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.

Number: Addition and Subtraction

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PROBLEM SOLVING					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = □ - 9	solve problems with addition and subtraction: * using concrete objects and pictorial representations, including those involving numbers, quantities and measures * applying their increasing knowledge of mental and written methods	solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction	solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why	solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why	solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
	solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change (copied from Measurement)				Solve problems involving addition, subtraction, multiplication and division

Number: Multiplication and Division

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MULTIPLICATION & DIVISION FACTS					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
count in multiples of twos, fives and tens (copied from Number and Place Value)	*count in steps of 2, 3, and 5 from 0, and in tens from any number, forward or backward* (copied from Number and Place Value)	*count from 0 in multiples of 4, 8, 50 and 100* (copied from Number and Place Value)	*count in multiples of 6, 7, 9, 25 and 1 000* (copied from Number and Place Value)	*count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000* (copied from Number and Place Value)	
	recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers	recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables	recall multiplication and division facts for multiplication tables up to 12 × 12		
MENTAL CALCULATION					
		write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods (appears also in Written Methods)	use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers	multiply and divide numbers mentally drawing upon known facts	perform mental calculations, including with mixed operations and large numbers
	show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot		recognise and use factor pairs and commutativity in mental calculations (appears also in Properties of Numbers)	multiply and divide whole numbers and those involving decimals by 10, 100 and 1000	*associate a fraction with division and calculate decimal fraction equivalents (e.g. 0.375) for a simple fraction (e.g. $^3/_8$)* (copied from Fractions)

Number: Multiplication and Division

National Centre for Excellence in the Teaching of Mathematics

WRITTEN CALCULATION					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs	write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods (appears also in Mental Methods)	multiply two-digit and three-digit numbers by a one-digit number using formal written layout	multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers	multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
				divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context	divide numbers up to 4-digits by a two-digit whole number using the formal written method of short division where appropriate for the context divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
					use written division methods in cases where the answer has up to two decimal places (copied from Fractions (including decimals))

Number: Multiplication and Division

National Centre for Excellence in the Teaching of Mathematics

PROPERTIES OF NUMBERS: MULTIPLES, FACTORS, PRIMES, SQUARE AND CUBE NUMBERS					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
			recognise and use factor pairs and commutativity in mental calculations (repeated)	identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.	identify common factors, common multiples and prime numbers *use common factors to simplify fractions; use common multiples to express fractions in the same denomination* (copied from Fractions)
				know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers	
				establish whether a number up to 100 is prime and recall prime numbers up to 19	
				recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})	*calculate, estimate and compare volume of cubes and cuboids using standard units, including centimetre cubed (cm^3) and cubic metres (m^3), and extending to other units such as mm^3 and km^3* (copied from Measures)

Number: Multiplication and Division

National Centre for Excellence in the Teaching of Mathematics

ORDER OF OPERATIONS					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
					use their knowledge of the order of operations to carry out calculations involving the four operations
INVERSE OPERATIONS, ESTIMATING AND CHECKING ANSWERS					
		estimate the answer to a calculation and use inverse operations to check answers (copied from Addition and Subtraction)	*estimate and use inverse operations to check answers to a calculation* (copied from Addition and Subtraction)		use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

Number: Multiplication and Division

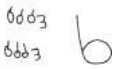
National Centre for Excellence in the Teaching of Mathematics

PROBLEM SOLVING					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher	solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts	solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects	solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects	solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes	solve problems involving addition, subtraction, multiplication and division
				solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign	
				solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates	*solve problems involving similar shapes where the scale factor is known or can be found* (copied from Ratio and Proportion)

The Bridge Federation Calculation Policy for addition: Year 1

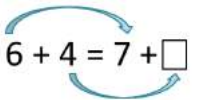
Mental Calculations

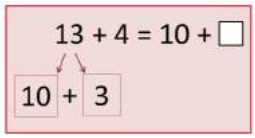
- **Read, write and interpret mathematical statements using symbols +, -, =**
- **Represent and use number bonds and related addition facts within 20**
- **Add one digit and two-digit numbers up to 20, including zero.**
- **Solve one-step problems using concrete objects and pictorial representations, and missing number problems such as 7 = □ - 9**
- **Given a number, identify (and use the language) one more**



Written Calculations

- Begin to compare (what's the same/different?) for commutative sums e.g 3 +7 = 7 + 3
- Understand 'part whole' models (see below) to identify the numbers 'hidden' within a number
- Use knowledge of 'part whole' to bridge across 10.
- *Memorise and reason with number bonds to 10 & 20 in several forms*
- Add using objects, Numicon, cubes etc and number lines and tracks
- Check with everyday objects
- Teach equality and inequality alongside each other
- Ensure pre-calculation steps are understood, including:
  - Counting objects (including solving simple concrete problems
  - Conservation of number:
  - **Recognise place value in numbers beyond 20**
  - **Counting** as reciting and as enumerating



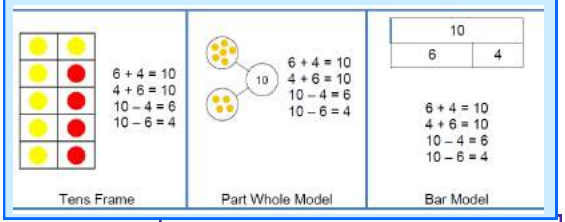


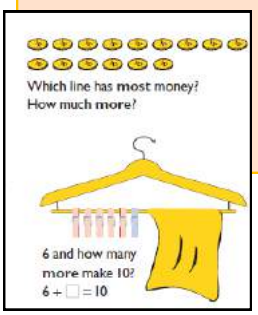
Representations to support mental and written calculations.

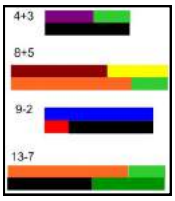
Use a range of concrete and pictorial representations, including:

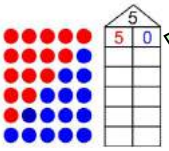
Using concrete and visual representations to show that 5 is greater than 2. Balance scales can help too.

'Part whole'
Making connections between models to help children understand the same maths represented in different ways.







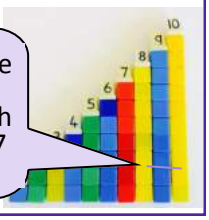


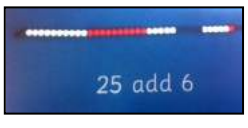
Shanghai Textbook Grade 1 (aged 6/7)

Can you see a pattern with the number bonds for 5?

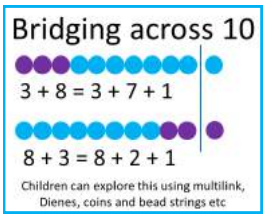
Staircase:

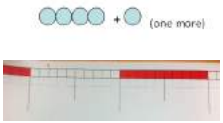
Can you see the numbers 'hidden' in each step? e.g. 2 + 7 inside 9



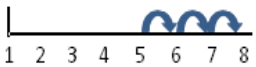


Bead strings





Number tracks



Number lines

Real everyday objects

Links from other strands

- *Combine and increase numbers, counting forwards and backwards.*
- *Develop the concept of addition and subtraction and … use these operations flexibly.*
- *Discuss and solve problems in familiar practical contexts, including using quantities*
- *Compare, describe and solve practical [measure] problems e.g. longer, more than, heavier than*
- Problems terminology should include: put together, add, altogether, total, take away, distance between, difference between, more than and less than.

The Bridge Federation Calculation Policy for addition: Year 2

Mental Calculations

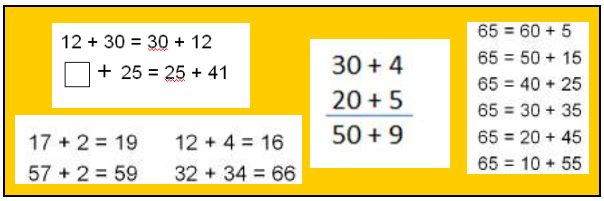
Add numbers using concrete objects, pictorial representations, and mentally, including:

- **a two-digit number and ones**
- **a two-digit number and tens**
- **two two-digit numbers**
- **adding three one-digit numbers**

- **Recall and use addition addition and subtraction facts to 20 facts fluently, and derive and use related facts up to 100**

Written Calculations

- **Demonstrate the commutative law of addition**
- Re-partition numbers eg. 65 (across page)
- Bridge across tens using knowledge of 'part whole'
- Use a hundred square
- *Check calculations using inverse and by adding numbers in different order*
- Begin to *record addition in columns to support place value and prepare for formal written methods with larger numbers*



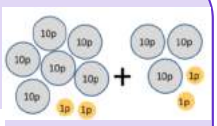
Representations to support mental and written calculations.

Use a range of concrete and pictorial representations, including:

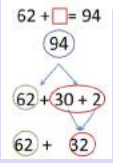
I know that 7 is greater than 6, so 5 plus 7 must be greater than 5 plus 6

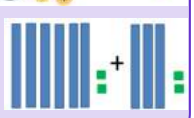
Teaching equality/inequality:
Use examples that children can reason about without the need to calculate e.g.
5 + 7 □ 5 + 6
True or false? 4 + 6 + 8 > 3 + 7 + 9

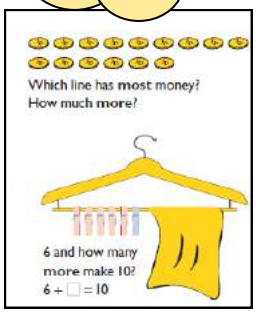
Unitising in 10s



60 + 2 = 62
62 + 2 = □
62 + 12 = □
62 + 22 = □



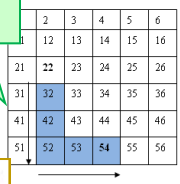


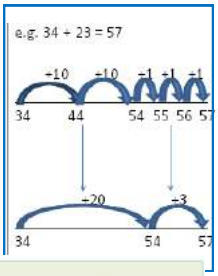


Use questioning to develop reasoning e.g.
What's the same? what different?

23 + 10	23 + 20	23 + 30	23 + 40

How can I use a 100 square to add 32 + 22?



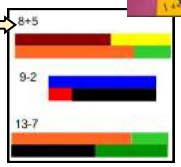


I can use Dienes to balance this number sentence

23 + 6 = 20 + □

Number tracks

Cuisennaire is a useful concrete resource that develops understanding of the pictorial bar model



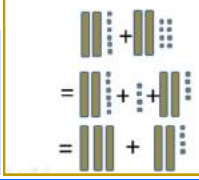
94	
62	32

'Magic 10'
19 + 16
= 19 + 1 + 15
= 20 + 15
= 35

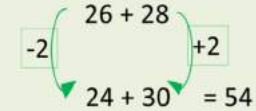
Bridging across tens

26 + 28:

- 26 + 4 + 24 = 30 + 24 = 54

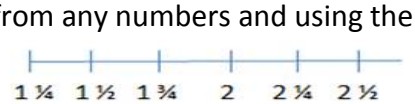


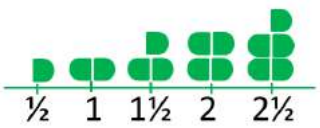
Compensating using balance:



Fractions

Counting in fractions up to 10, starting from any numbers and using the ½ and $^{2}/_{4}$ equivalence on the number line





Links from other strands

- Solve problems:
- Using concrete objects, pictorial representations (numbers, quantities & measures)
- Applying increasing knowledge of mental & written methods
- Partition numbers in different ways
- *Discuss and solve problems that emphasise the value of each digit in two-digit numbers*

The Bridge Federation Calculation Policy for addition: Year 3

Mental Calculations

Add numbers mentally, including:

- **a three-digit number and ones**
- **a three-digit number and tens**
- **a three digit number and hundreds**
- Partition all numbers and recombine, start with TU + TU then HTU + TU
  Bridge across tens and hundress using 'part whole'
- Use straws, dienes, place value counters, coins empty number lines

Common mental calculation strategies:
Partitioning and recombining
Doubles and near doubles
Use number pairs to 10 and 100
Adding near multiples of ten and adjusting
Using patterns of similar calculations
Using known number facts
Bridging though ten, hundred
Complementary addition

Written Calculations

Add numbers with up to three digits, using formal written (columnar) methods

Add to three digit numbers using physical and abstract representations (e.g. straws, dienes, place value counters, empty number lines, coins)
Use manipulatives to support structure of the algorithm especially place value

30 + 4	34
20 + 5	+25
50 + 9	59

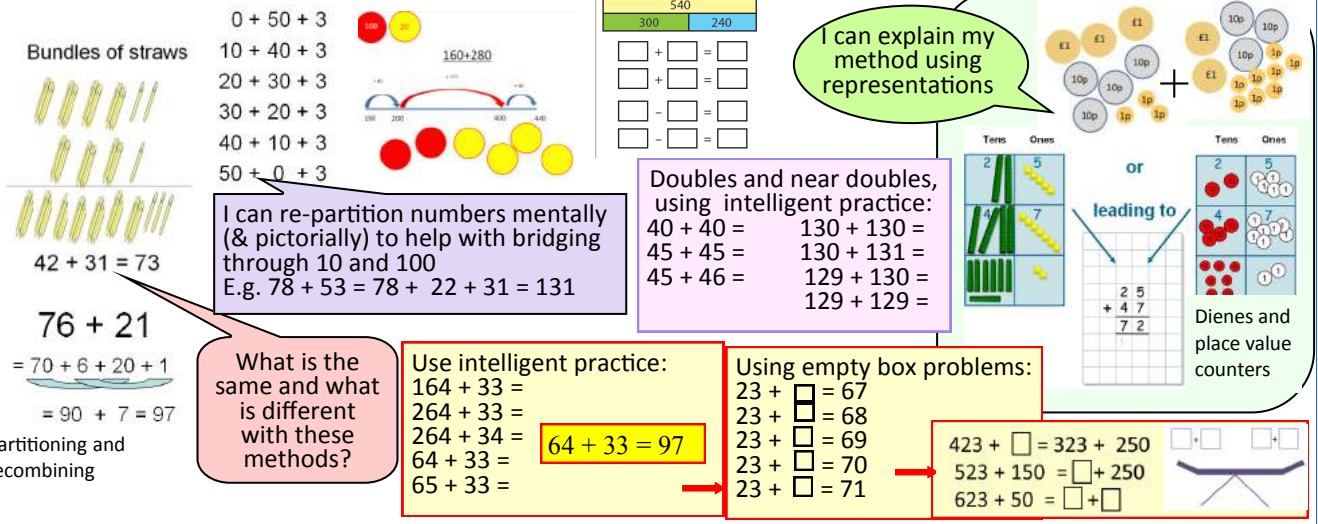
200 + 30 + 4	234
500 + 20 + 7	+ 527
700 + 60 + 1	761
10	1

Informal methods of recording are used as stepping stones to help children understand the logic of formal written methods.

Revert to concrete representations if children find expanded/column methods difficult

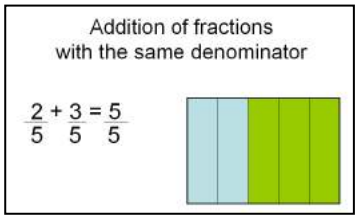
Representations to support mental and written calculations.

Use a range of concrete, pictorial and abstract representations, including those below



Fractions

Addition of fractions with the same denominator within one whole.



Links from other strands

Pupils should estimate the answers to a calculation & use inverse operations to check answers.
Add amounts of money using both £ and p in practical contexts.
Measure, compare and add lengths (m/cm/mm), mass (kg/g) & volume/capacity (l/ml)

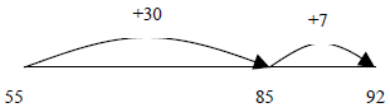
The Bridge Federation Calculation Policy for addition: Year 4

Informal methods to support mental Calculations

Practise mental methods with increasingly large numbers

Consolidate partitioning and re-partitioning
Bridge tens and hundreds using partitioning and 'part whole'
Use compensation for adding too much/little and adjusting
Use straws, Dienes, place value counters,
empty number lines etc.

I know that 63 + 29 is the same as 63 + 30 –1



55 + 37 = 55 + 30 + 7
= 85 + 7
= 92

Common mental calculation strategies:
Partitioning and recombining
Doubles and near doubles
Use number pairs to 10 and 100
Adding near multiples of ten and adjusting
Using patterns of similar calculations
Using known number facts
Bridging though ten, hundred
Complementary addition

Written Calculations

Add numbers with up to four digits, using the formal written (columnar) method

Add three digit numbers using columnar method and then move onto 4 digits.
Include decimal addition for money
Use manipulatives to support structure of the algorithm especially place value

789 + 642 becomes

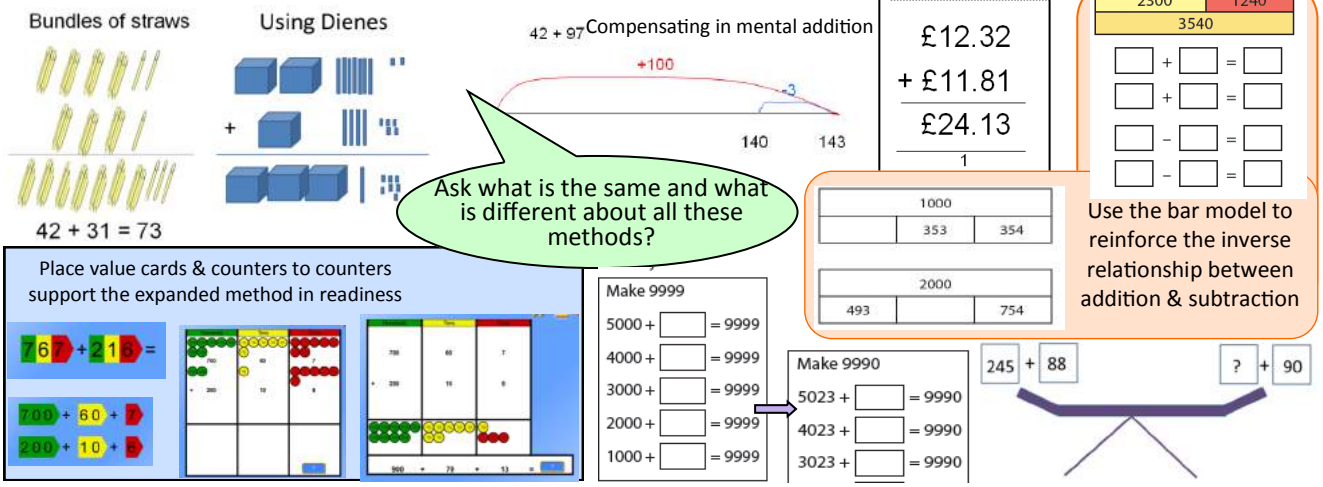
	7	8	9
+	6	4	2
1	4	3	1
	1	1	

Answer: 1431

Revert to expanded methods if children find formal calculation method difficult

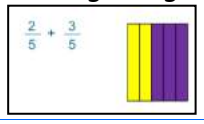
Representations to support mental and written calculations.

Use physical/pictorial representations alongside expanded and columnar methods.

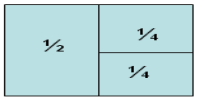


Fractions

Addition of fractions with the same denominator *to become fluent through a variety of increasingly complex problems beyond one whole*
Counting using simple fractions and decimals, both forwards and backwards



$$\frac{1}{2} + \frac{2}{4} = \frac{2}{4} + \frac{2}{4} = 1$$



Links from other strands

- Estimate and use inverse operations to check answers.
- Solve addition and subtraction two step problems in context, deciding which operations and methods to use and why
- Identify, represent and estimate numbers using different representations. (Place value)
- Recognise the place value of each digit in a four-digit number.
- Estimate, compare and calculate different measures, including amounts money in £ and p (including fractions and decimals)

The Bridge Federation Calculation Policy for addition: Year 5

Informal methods to support mental Calculations

- **Add numbers mentally with increasingly large numbers, e.g. 12 462 + 2300 = 14 762**
- **Mentally add tenths, and one-digit numbers and tenths**
- **Add decimals, including a mix of whole numbers and decimals, decimals with different numbers of places, and complements of 1 (e.g. 0.83 + 0.17 = 1)**

Children use representation of choice
Refer back to pictorial and physical representations when needed.
Use concept of balance/equivalence to compensate
Bridge across boundaries by partitioning

Common mental calculation strategies:
Partitioning and recombining
Doubles and near doubles
Use number pairs to 10 and 100
Adding near multiples of ten and adjusting
Using patterns of similar calculations
Using known number facts
Bridging though ten, hundred, tenth
Complementary addition

Written Calculations

Add whole numbers with more than four digits, using the formal written (columnar) method

Add three digit numbers using columnar method and then move onto 4 digits.
Include decimal addition for money
Use manipulatives to support structure of the algorithm especially place value

```
  24172m
+  5929m
  30101m
  1 1 1 1
```

```
  £563.14
+ £207.88
  £771.02
    1 1 1
```

Revert to expanded methods if children find formal calculation method difficult (see Y3)

Represent-ations to support mental and written calculations.

Use physical/pictorial representations alongside columnar methods where needed.

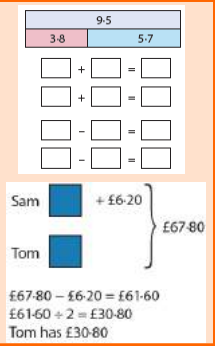
12 462 + 2300
= 12 462 + 2000 + 300
= 14 462 + 300
= 14 762

Partitioning and recombining

Jottings to support mental calculation

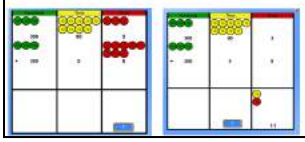
Use the **bar model** to reinforce the inverse relationship between addition & subtraction:

This supports problem solving: Sam and Tom have £67.80 between them. If Sam has £6.20 more than Tom, how much does Tom have?



1.6 + 1.4 = 3
Write down three more pairs of decimal numbers that sum to 3

Place Value counters to support column addition



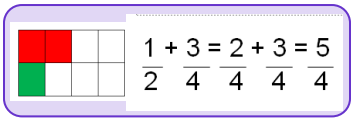
```
  393
+ 308
  1
  1
```

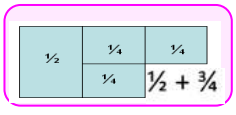
Compensating: true or false?
2741 + 1263 = 2742 + 1262
Why? Can you use resources or draw a picture to explain your answer? How can you adjust this to make the calculation easier? 3498 + 2067

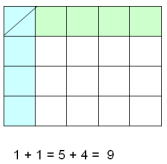
What is the same and **what is different** about all these methods?

Fractions

- **Add fractions with the same denominator and denominators that are multiples of the same number** *(to become fluent through a variety of increasingly complex problems and add fractions that exceed 1 as a mixed number)*







Links from other strands

- Solve problems involving up to three decimal numbers.
- Solve addition and subtraction multi step problems in context, deciding which operations and methods to use and why
- Use all four operations to solve problems involving measure [e.g. length, mass, volume, money] using decimal notation,
- Calculate the perimeter of composite rectilinear squares in centimetres and metres
- Use angle sum facts and other properties to make deductions about missing angles
- Solve comparison, sun and difference problems using information presented in a line graph

The Bridge Federation Calculation Policy for addition: Year 6

Informal methods to support mental Calculations

- **Perform mental calculations, including with mixed operations and large numbers** *(more complex calculations)*

Children use representation of choice
Consolidate partitioning and re-partitioning for bridging boundaries (tens, hundreds, thousands, tenths, hundredths …)
Use compensation for adding too much/little and adjusting
Refer back to pictorial and physical representations when needed.
Apply the rules of BIDMAS

Common mental calculation strategies:
Partitioning and recombining
Doubles and near doubles
Use number pairs to 10 and 100
Adding near multiples of ten and adjusting
Using patterns of similar calculations
Using known number facts
Bridging though ten, hundred, tenth
Complementary addition

Written Calculations

Add larger numbers using the formal written (columnar) method

Add three digit numbers using columnar method and then move onto 4 digits.
Include decimal addition for money

£563.14
+ £207.88
£771.02
1 1 1

789 + 642 becomes

	7	8	9
+	6	4	2
1	4	3	1
	1	1	

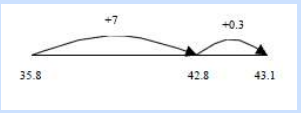
Answer: 1431

Revert to expanded methods if children find formal calculation method difficult (see Y3)

Representations to support mental and written calculations.

Use physical/pictorial representations alongside columnar methods where needed. Ask what is the same and what is different?

12 462 + 2300
= 12 462 + 2000 + 300
= 14 462 + 300
= 14 762



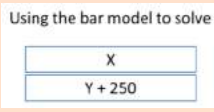
Jottings to support mental strategies

234 kg + 49 kg = 273 kg

200 + 30 + 4
40 + 9
200 + 70 + 13

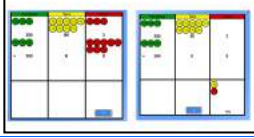
x and y represent whole numbers.
Their sum is 1000.
x is 250 more than y.
What are the values of x and y?

Using the bar model to solve problems



I can explain my method using place value counters

Place Value counters to support column addition



393
+ 308
1
1

Follow the BIDMAS order of operations!
Brackets
Indices (powers of e.g. 2^2)
Division
Multiplication
Addition
Subtraction

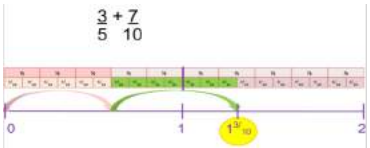
Compare 31 + 9 × 7 and (31 + 9) × 7
What's the same? What's different?

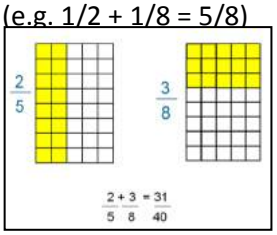
14 781 – 6□53 = 8528
23·12 + 22·□= 45·23

Can you use five of the digits 1 to 9 to make this number sentence true?
□□·□ + □·□ = 31·7
Can you find other sets of five of the digits 1 to 9 that make the sentence true?

Fractions

- **Add fractions with different denominators and mixed numbers, using the concept of equivalent fractions**
- Start with fractions where the denominator of one fraction is a multiple of the other (e.g. 1/2 + 1/8 = 5/8) and progress to varied and increasingly complex problems
- Practise calculations with simple fractions and decimal equivalents to aid fluency





Links from other strands

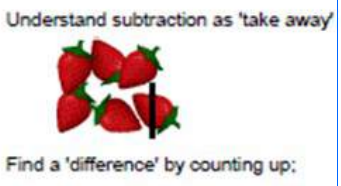
- Use their knowledge of the order of operations to carry out calculations involving the four operations (BIDMAS)
- Solve problems involving all four operations
- Algebra: use symbols and letters to represent variable and unknowns *e.g. a + b = b + a*
- Solve problems involving the calculation and conversions of units of measure, using decimal notation of up to three decimal places where appropriate
- *Using the number line, pupils use, add and subtract positive and negative integers for measures such as temperature*
- Calculate and interpret the mean as an average
- Interpret and construct pie charts and line graphs and use these to solve problems
- Find missing angles, *and express geometry relationships algebraically (e.g. d=2xr)*

The Bridge Federation Calculation Policy for subtraction Year 1

Mental Calculations

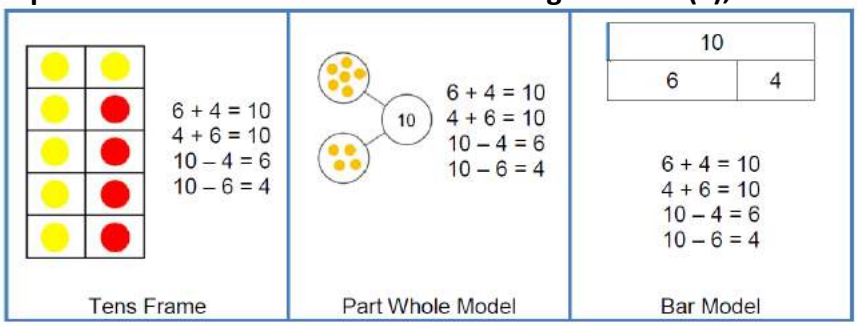
Subtract one digit and two-digit numbers to 20, including zero.
Read, write and interpret mathematical statements using symbols (+, -, =) signs.
Represent and use number bonds and related addition facts within 20
Solve one-step problems using concrete objects and pictorial representations, and missing number problems such as 7 = - 9
Memorise and reason with number bonds
Add using objects, Numicon, cubes etc. and number lines and tracks
Check with everyday objects
Ensure pre-calculation steps are understood, including:
Counting objects,

Conservation of number



Written Calculations

Subtract one-digit and two-digit numbers to 20, including zero.
Read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs .



Represent and use number bonds and related subtraction facts within 20.

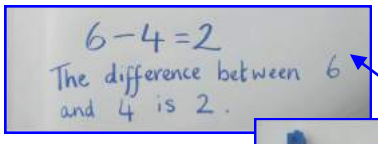
Representations to support mental and written calculations.

Use a range of concrete and pictorial representations, including:

Hands, fingers and children themselves.

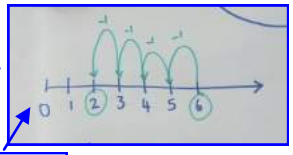
Straw bundles

Bead strings, number tracks and lines



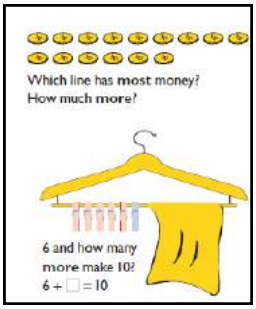
Difference or comparison model

Counting on or back.

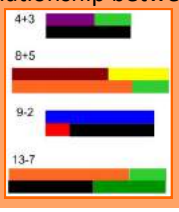


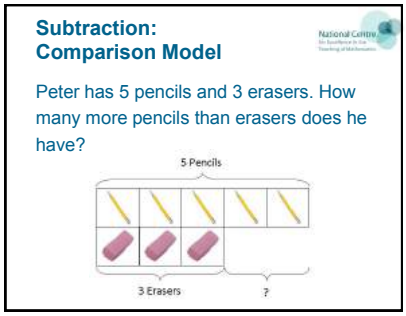
6 - 4= □

Part Whole



Models such as Cuisennaire or balances reinforce the relationship between addition and subtraction.





Links from other strands

Pupils should combine and increase numbers, counting forwards and backwards.
(They should) develop the concept of addition and subtraction and … use these operations flexibly.
Problems should include the terms: put together, add, altogether, total, take away, distance between, difference between, more than and less than, so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.
(Number-addition and subtraction, Non-statutory guidance.)
Pupils discuss and solve problems in familiar practical contexts . (*Non-statutory guidance.)*
Pupils compare, describe and solve practical (measurement) problems .
(Measurement)

The Bridge Federation Calculation Policy for subtraction Year 2

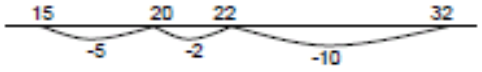
Mental Calculations

Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:

- **a two-digit number and ones**
- **a two-digit number and tens**
- **two two-digit numbers**
- **adding three one-digit numbers**

Jottings to support informal methods:

Bridge through 10 where necessary
32 - 17

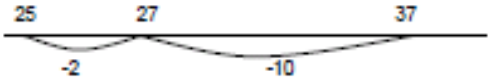


54—32 = 22

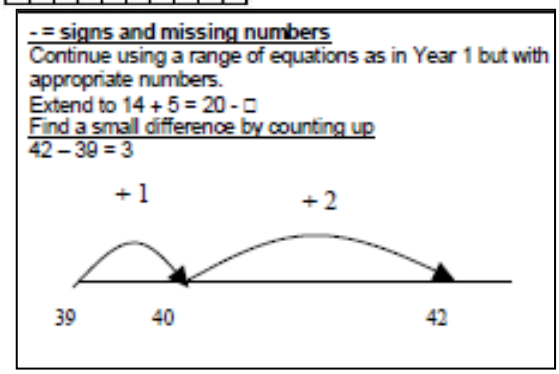
Written Calculations

Written recording:

37 – 12 = 37 – 10 - 2
= 27 – 2
= 25



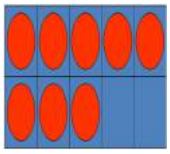
Continue to use of a range of concrete and pictorial representations from Year 1, including Bar model to support understanding of **difference.** (See below).



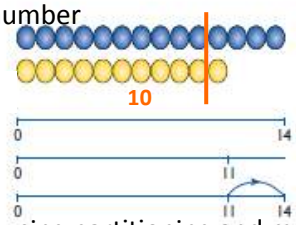
Representations to support mental and written calculations.

Informal methods to support written subtraction calculations

Practical partitioning of a 2-digit number



In Year 1 leads to:



The difference between 11 and 14 is 3.
14 – 11 = 3
11 + □ = 14

which can lead to exploration and variation

4 - 1 = 3
14 -11 = 3
24 -21 = 3

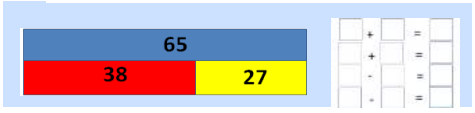
Subtract (without decomposition) using partitioning and manipulatives, e.g. Dienes or straw bundles

To calculate 35-22, remove 22.

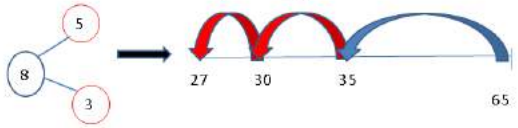
Then record: **35-22=13**.

Pupils experience bridging through 10 using number bonds and the Part Whole model.

65 – 38 = ...



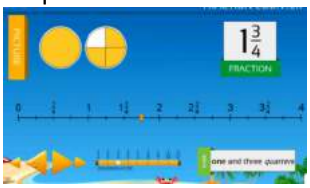
8 can be partitioned using the Part Whole model.



Fractions

Pupils should count in fractions up to 10, starting from any number and using the and equivalence on the number line (for example, 1 ¼, 1 ½, 1 ¾, 2.)

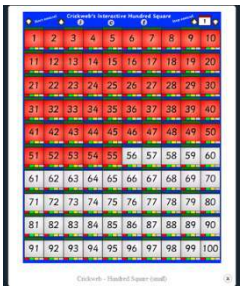
Use concrete and pictorial models of fractions to assist with counting e.g. paper cups, plates, shapes etc.



Links from other strands

Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100.

Pupils should partition numbers in different ways (for example, 23 = 20 + 3 and 23 = 10 + 13) to support subtraction.



55 + 45 = 100
45 + 55 = 100
35 + 65 = 100
100—55 = 45
100—45 = 55
100—35 = 65

Solve problems with addition and subtraction:

- **using concrete objects and pictorial representations, including those involving numbers, quantities and measures**
- **applying their increasing knowledge of mental and written methods**
- ***Pupils extend their understanding of the language of addition and subtraction to include sum and difference.***

The Bridge Federation Calculation Policy for subtraction Year 3

Mental Calculations

Add and subtract numbers mentally, including:
***a three-digit number and ones**
***a three-digit number and tens**
***a three-digit number and hundreds.**

101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

247	
173	74

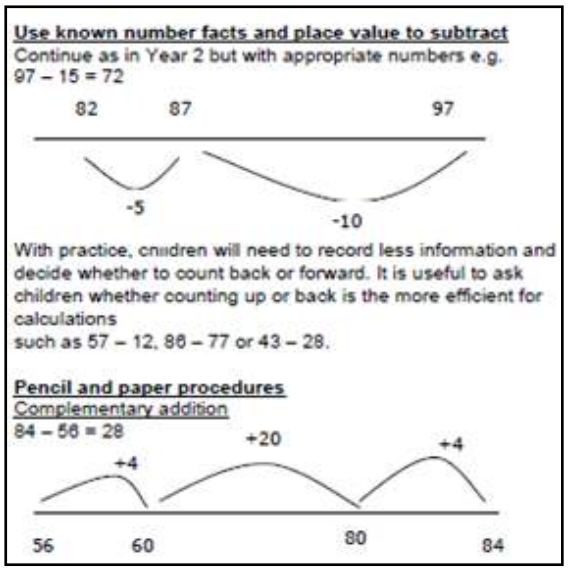
173 + 74 = 247

74 + 173 = 247

247 – 173 = 74

247 – 74 = 173

Use a number line, Dienes, 100 squares, 200 hundred squares, and similar representations, to support mental calculations. (See below.)



Written Calculations

Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.

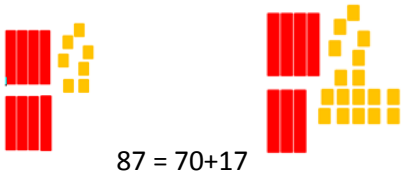
(1)Extended columnar - no exchange

Extended method 87 - 53 =

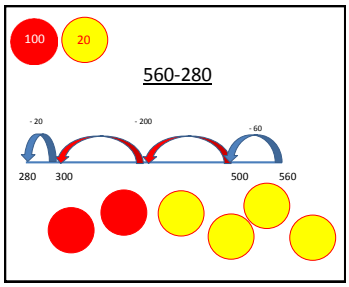
80 and 7
- 50 and 3
30 and 4 = 34

(2) Extended columnar – with exchange:
87-58 becomes

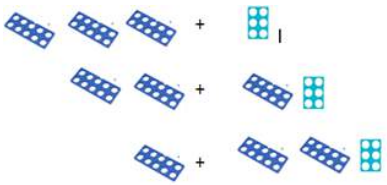
70 +1 7
-50 + 8
20 + 9



Representations to support mental and written calculations.



Partitioning and re- partitioning support the understanding of place-value.



30 + 6
20 + 16
10 + 26

All of these representations still comprise the amount of 36.

Introduce transition from concrete place value representations, (e.g. dienes or straws), to pictorial – such as place value counters or money.

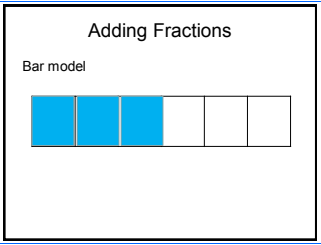
132 in Dienes 132 in place value counters.

Revert to concrete manipulatives and expanded methods whenever difficulties arise

Fractions

Count up and down in tenths.
Add and subtract fractions with the same denominator within one whole.

$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$



Links from other strands

Money and calculating duration of events (with number lines.)

For example: ***"Add and subtract amounts of money to give change, using both £ and p in practical contexts."***

"Compare durations of events [for example to calculate the time taken by particular events or tasks]." (Measurement)

The Bridge Federation Calculation Policy for subtraction Year 4

Mental Calculations

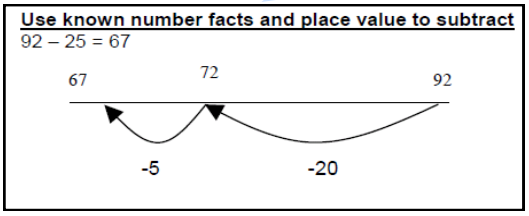
Continue to practise mental methods with increasingly large numbers to aid fluency. *(From Non– Statutory Guidance).*

Methods to support fluent calculation and encourage efficiency of method:

- Find a small difference by counting up.
  E.g. 5003—4996
- Subtract nearest multiple of ten and adjust.
- Partition larger numbers

This could be done using an empty number line. Children should recall and use number facts to reduce the number of steps.

Whenever possible, children should be encouraged to visualise number lines and other basic, supporting representations to promote fluent work without jottings.



Written Calculations

Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate.

Build on formal, extended method (*See Year 3)* using exchange wherever necessary.

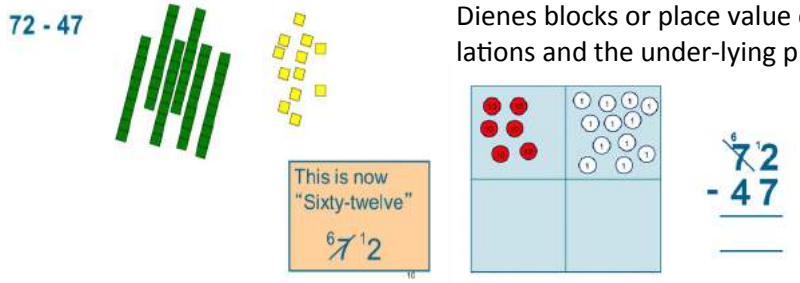
Continue to use representations and manipulatives to develop understanding of place value.

372—147 =

$$\begin{array}{r} 300 + 70 + 2 \\ -100 + 40 + 7 \end{array} \rightarrow \begin{array}{r} 300 + 60 + 12 \\ -100 + 40 + 7 \\ \hline 200 + 20 + 5 \end{array} \rightarrow \begin{array}{r} {}^{60}\not{3}00 + \not{7}0 + {}^{1}2 \\ -\ 100 + 40 + 7 \\ \hline 200 + 20 + 5 \end{array}$$

Apply understanding of subtraction with larger integers to that of decimals in context of money and measures. (See Year 5.)

Representations to support mental and written calculations.

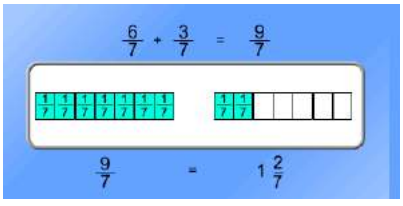


Dienes blocks or place value counters can be used to model calculations and the under-lying place value concepts.

Use physical and / or pictorial representations and expanded algorithms alongside columnar methods. Ask: *What is the same? What's different?*
Compare and discuss the suitability of different methods in context.
Pupils **decide which operations and methods to use and why.**

I would count on using a number line to calculate 5003-4896; because the numbers are close together.

Fractions



Count up and down in hundredths.
Add and subtract fractions with the same denominator .
Solve simple measure and money problems involving fractions and decimals to two decimal places.

Links from other strands

Identify, represent and estimate numbers using different representations. (*Place value)*
Recognise the place value of each digit in a four-digit number.
Estimate and use inverse operations to check answers to a calculation .
Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.
Estimate, compare and calculate different measures, including money in pounds and pence.

The Bridge Federation Calculation Policy for subtraction Year 5

Mental Calculations

- **Subtract numbers mentally with increasingly large numbers.**
  E.g. 12 462 – 2300 = 10 162
- **Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy .**
- ***Pupils practise adding and subtracting decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1 (for example, 1 - 0.17 = 0.83).***
- ***Pupils mentally add and subtract tenths, and one-digit whole numbers and tenths.***

Basic Mental Strategies for Subtraction

- Find differences by counting up
- Partitioning
- Applying known facts
- Bridging through 10 and multiples of 10
- Subtracting 9, 11 etc. by compensating
- Counting on to, or back from the largest number

National Curriculum 1999

Which method works best? Why? How else could we do it?

Children use, or visualise, representation of choice. Refer back to physical representations as required.

Written Calculations

Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction).
(Pupils) practise adding and subtracting decimals.
Begin with three-digit numbers using formal, columnar method; then move into four-digit numbers.

As in Year 4, compare physical and / or pictorial representations and expanded algorithms alongside columnar methods. Ask: *What is the same? What's different?*
Compare and discuss the suitability of different methods, (mental or written), in context.
Revert to expanded methods whenever difficulties arise

£17.34—£12.16

What is the same about these models? What's different?

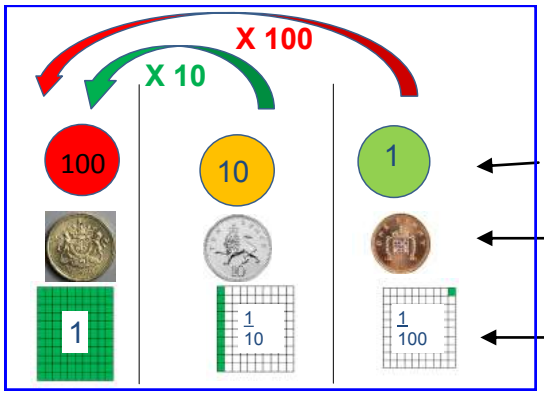
1000+700+20+14p
- 1000+200+10+ 6p
500+10+ 8p

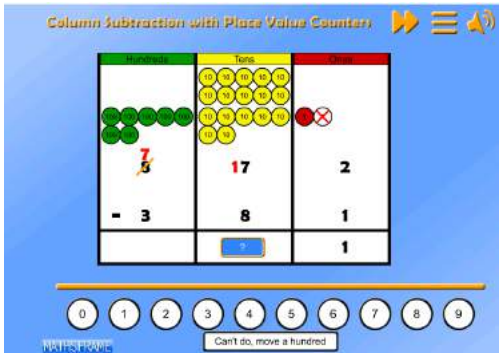
→ 2 1734p - 1216p 518p →

£ 2 17.34 - 12.16 5.18

Relate place value of decimals with that of whole numbers using representations. See below.

Representations to support mental and written calculations.





Use physical and pictorial representations to stress the place value relationships between money, decimals and whole numbers. A place value mat such as the this one could be used, moving away from the traditional: *Hundreds, tens and ones* model used in Lower KS2 and KS1.

Fractions

Subtract fractions with the same denominator and denominators that are multiples of the same number. (*Include fractions exceeding 1 as a mixed number.)*
Solve problems involving number up to three decimal places .
They mentally add and subtract tenths, and one-digit whole numbers and tenths.

Links from other strands

Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign.
Use all four operations to solve problems involving time, money and measure using decimal notation.; (up to 3d.p.)

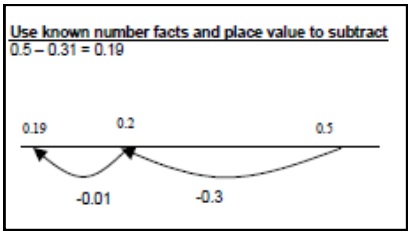
The Bridge Federation Calculation Policy for subtraction Year 6

Mental Calculations

Children:

- **Perform mental calculations, including with mixed operations and large numbers.**
- **Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.**
- ***They undertake mental calculations with increasingly large numbers and more complex calculations.***

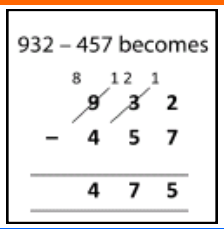
Children draw on basic, Mental subtraction Strategies, (See Year 5.)
Children use, or visualise, representation of choice.
Refer back to physical representations as required.



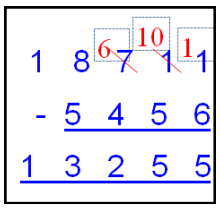
Written Calculations

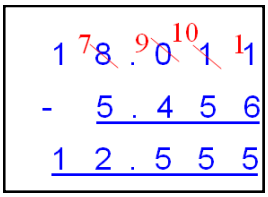
Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction). Solve problems involving the calculation and conversions of units of measure, using decimal notation of up to three decimal places where appropriate. (MEASURES)

Move towards consolidation of formal, columnar method.
For more complex calculations, with increasingly larger or smaller numbers, compare representations and expanded algorithms alongside columnar methods. Ask: What is the same? What's different?
Compare and discuss the suitability of different methods, (mental or written), in context.
Revert to expanded methods whenever difficulties arise

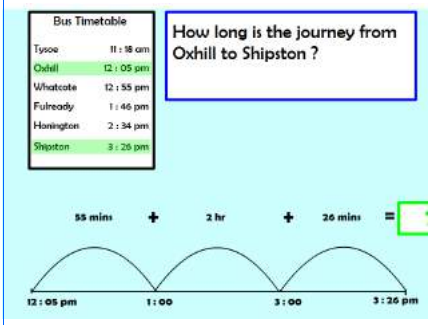


Consolidate columnar methods , paying particular attention to the occurrence of zeros as place holders.

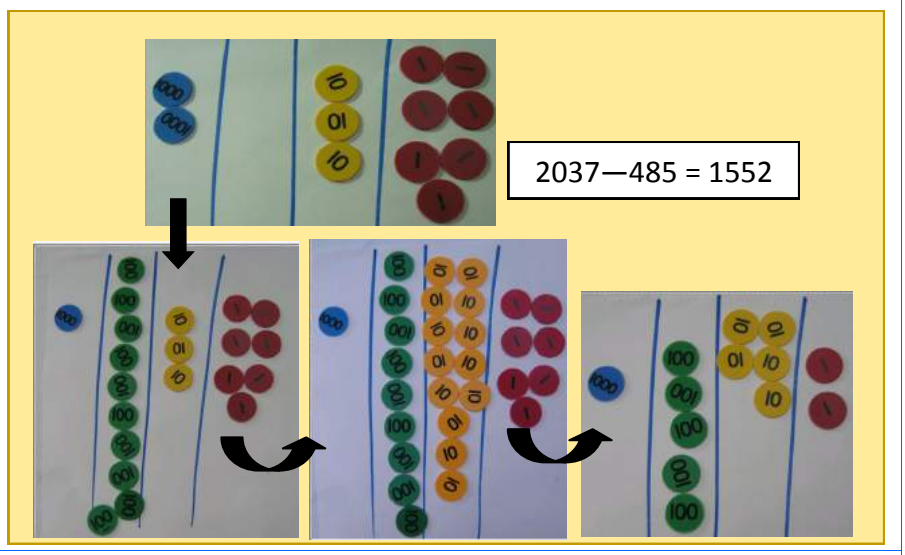




Representations to support mental and written calculations.



Use physical/pictorial representations alongside columnar methods where needed. *What is the same, what is different?*



Fractions

Add and subtract fractions with different denominators and mixed numbers.
They practise calculations with simple fractions and decimal fraction equivalents to aid fluency.

Links from other strands

Use their knowledge of the order of operations to carry out calculations involving the four operations (BIDMAS)
Solve problems involving all four operations
Algebra: use symbols and letters to represent variable and unknowns e.g. a + b = b + a
Using the number line, pupils use, add and subtract positive and negative integers for measures such as temperature.

The Bridge Federation Calculation Policy for multiplication: Year 1

Mental Calculations

- **solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.**
- **Count in multiples of twos, fives and tens** with equipment, songs & rhythms, and including by rote
  - Counting 2s e.g. counting socks, shoes, animal legs...
  - Counting in 5 s e.g. counting fingers, fingers in gloves, toes …
  - Counting in 10s e.g. counting fingers, toes ...
- Doubles up to 10
- Recognising odd and even numbers
- Write as a number pattern (e.g. 5, 10, 15...; 2, 4, 6...; 10, 20, 30...)

What's the sequence?

What comes next?

Written Calculations

It is important to use a range of models to develop understanding of multiplication, and that children make connections between arrays, number patterns, and counting in twos, fives and tens

Although there is no statutory requirement for written multiplication in Year 1, it may be helpful to encourage children to begin to write it as a repeated addition sentence in preparation for Year 2
E.g. 2 + 2 + 2 + 2 = 8

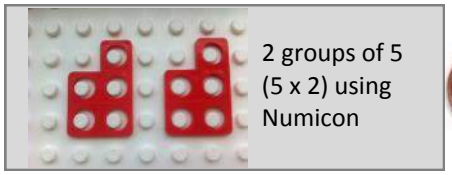
Plan for short daily focus sessions on number facts

Representations to support mental and written calculations.

Use a range of concrete and pictorial representations, including:

There are 3 sweets in one bag.
How many sweets are there in 5 bags?

4 groups of 3
3 groups of 4



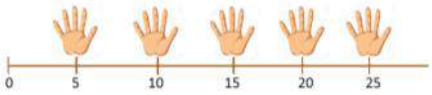
2 groups of 5
(5 x 2) using Numicon

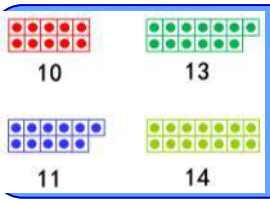
4 groups of 2p
2p multiplied by 4
2p x 4 = 8p

Can I use doubling?

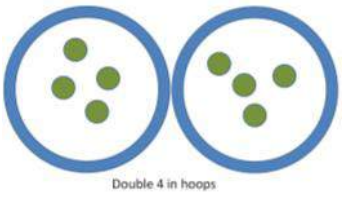
4 x 3 = 12
"4 cakes, 3 times"
4 multiplied by 3

Concrete to pictorial: counting in 5s

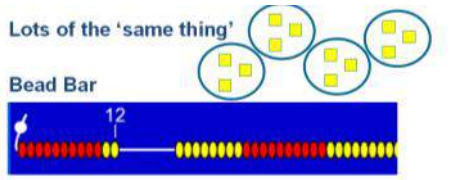


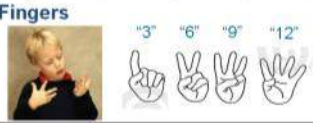


What do you notice about odd numbers?



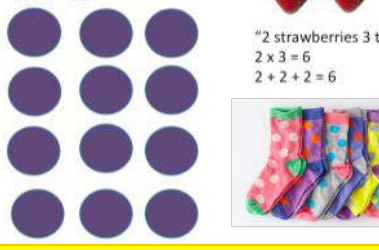
Double 4 in hoops





Anna is counting in fives:
5, 10, ☐, 20, ☐, ☐, …

3 + 3 + 3 + 3 = 12
3 multiplied by 4 is 12
3 x 4 = 12



"2 strawberries 3 times"
2 x 3 = 6
2 + 2 + 2 = 6

Contextualise the mathematics:
Susie invites 6 friends to her birthday party.
How many cherries are there on the plate?
How many biscuits will we need if we eat 2 each?
There are 5 sweets for each party bag. How many sweets to I need altogether?

How do they count?
In 1s? 2s? 5s? 10s?

Links from other strands

- **Count in multiples of twos, fives and tens (**from Number and place value), as above
- *Counting in twos, five and tens from different multiples to develop their recognition of patterns in the number system*
- *They discuss and solve problems in familiar practical contexts, including using quantities.*

The Bridge Federation Calculation Policy for multiplication: Year 2

Mental Calculations

- **Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables,** *connecting the 2, 5 and 10 multiplication tables to each other*
- *Connect the 10 multiplication table to place value*
- **Recognise odd and even numbers**
- **show that multiplication of two numbers can be done in any order (commutative)**
- *Use a variety of language to describe multiplication and division*
- Apply doubling of numbers up to ten to doubling larger numbers

I know that the multiples of 2/5/10 are always/never ….

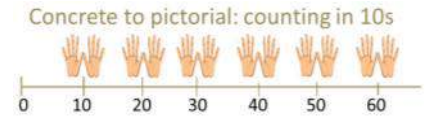
Written Calculations

- **calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs**
- *Begin to use other multiplication tables and recall facts to perform written calculations*
- *Use a range of materials and contexts … including arrays and repeated addition*

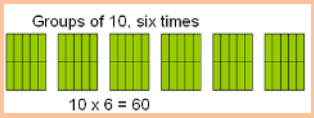
7 x 2 = □
7 x □ = 14
□ x 2 = 14
△ x □ = 14

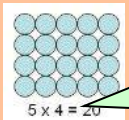
Representations to support mental and written calculations.

Use a range of concrete and pictorial representations, including:

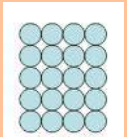


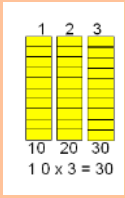
What multiplication sentences can you write with these numbers: 5, 10, 50?



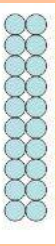


I want five, four times

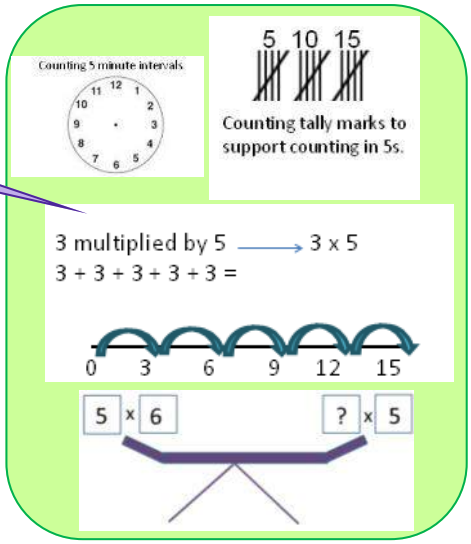


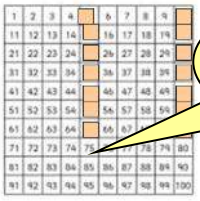


What arrays can you make with 20 counters?

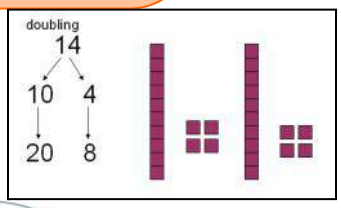


I want four, five times



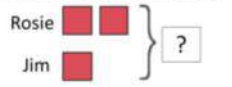


What do you notice about the numbers covered up? Is there a pattern? What number is next?

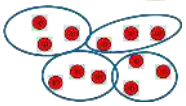


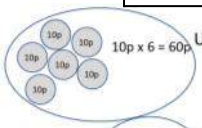
Using the **bar model** to solve problems

A book costs £5.
Rosie buys twice as many as Jim.
How much do they spend altogether?

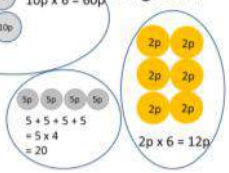


5 x 3 = 15
They spend £15 altogether.





Using coins:



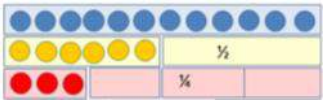
Develop an understanding of the equals sign:

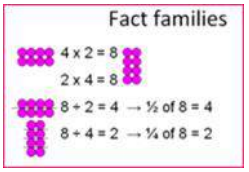
10 + 10 = 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2
5 + 5 + 5 + 5 = 4 + 4 + 4 + 4 + 4

Contextualise the maths:
Would you rather have:
4 packets of biscuits with 5 in each packet, or
3 packets of biscuits with 10 in each packet?
Explain your answer.

Fractions

- **write simple fractions for example, 1/2 of 6 = 3 and recognise the equivalence of two quarters and one half**
- *Begin to relate multiplication and division models to fractions and measures*





Links from other strands

- **solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.**
- *Use commutativity and inverse relations to develop multiplicative reasoning (e.g. 4 x 5 = 20 and 20 ÷ 5 = 4)*
- **Statistics—interpret and consttruct simple pictograms, tally charts and block diagrams**
- Measurement— coiunting 5 minute intervals on a clock face
- Place value **count in steps of 2, 3 and 5 from 0 and in tens from any number, forwards and backwards**

The Bridge Federation Calculation Policy for multiplication: Year 3

Mental Calculations

- **recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables** (and 2, 5 and 10 multiplication tables from Y2)
- Use doubling to connect 2, 4 and 8 multiplication tables
- Develop efficient mental methods using commutativity and associativity
- Derive related multiplication and division facts
- **calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods**
- Partitioning: multiply the tens first and then multiply the units,
  e.g. 57 x 6 = (50 x 6) + (7 x 6) = 300 + 42 = 342
- Children can apply these skills to solve spoken word problems too,
- Include missing number statements e.g $72 \div \square = 8$

The associative law:
4 x 12 x 5 = 4 x 5 12
= 20 x 12
= 240

The commutative law:
4 x 12 = 12 x 4

I have 8 packets, each containing 12 crayons. How many crayons do I have in total?'

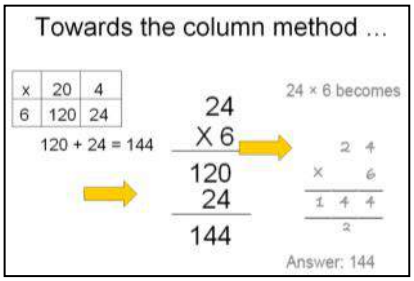
Ensure opportunities to learn multiplication tables through use of visual models, images and also rote learning.

Multiplication and division facts:
8 x 4 = 32, 4 x 8 = 32, 32 ÷ 4 = 8, 32 ÷ 8 = 4

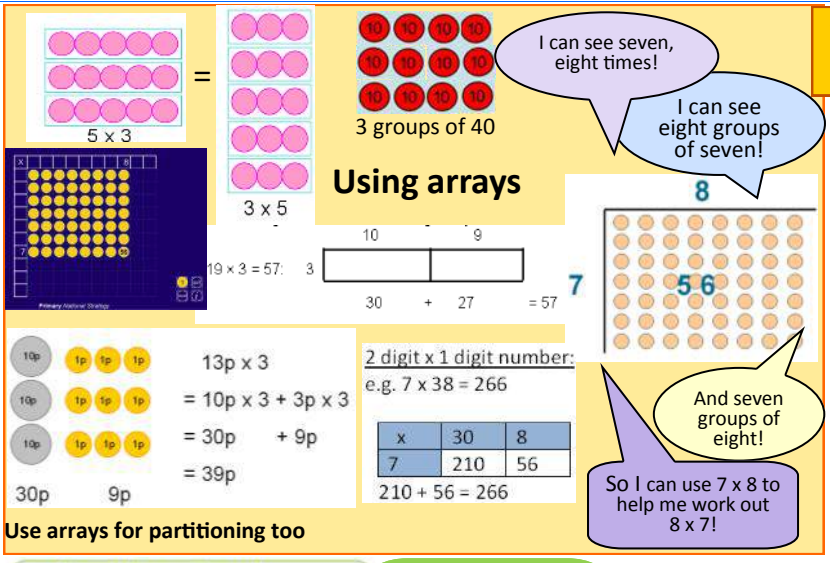
Deriving related facts:
3 x 2 = 60, 6 ÷ 3 = 2, 6 ÷ 2 = 3
➡ 30 x 2 = 60, 60 ÷ 3 = 20, 20 = 60 ÷ 3

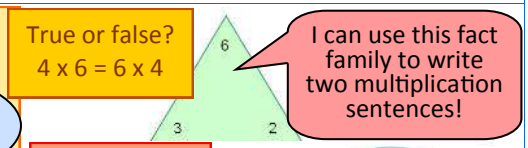
Written Calculations

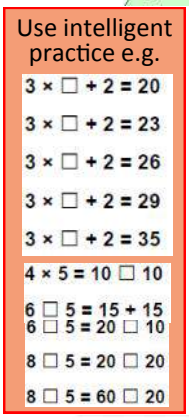
- **write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, progressing to formal written methods**
- Estimate before calculating
- Ensure written methods build on/relate to mental methods

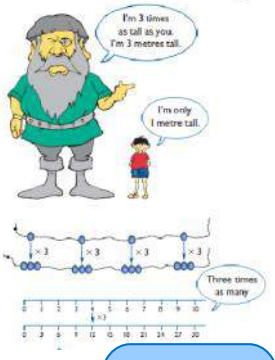


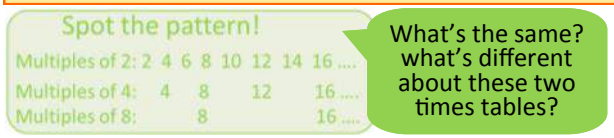
Representations to support mental and written calculations.

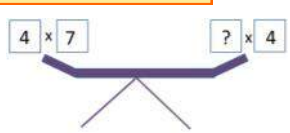


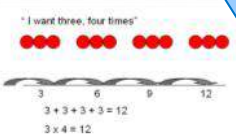








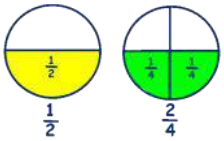


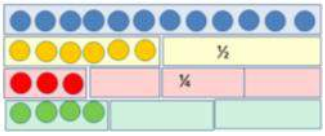


Which is the odd one out? Why?
24 x 3
36 x 4
13 x 5
32 x2

Fractions

- **recognise and show, using diagrams, equivalent fractions with small denominators**





Links from other strands

- **solve problems, including missing number problems, involving multiplication , including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.**
- *The comparison of measures includes simple scaling by integers (for example, a given quantity or measure is twice as long or five times as high)*
- *Pupils now use multiples of 2, 3, 4, 5, 8, 10, 50 and 100.*
- Pupils understand and use simple scales (for example, 2, 5, 10 units per cm) in pictograms and bar charts with increasing accuracy.

The Bridge Federation Calculation Policy for multiplication: Year 4

Informal methods to support mental Calculations

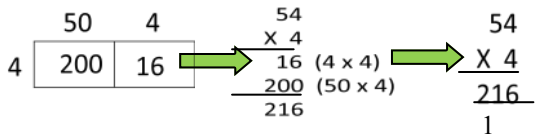
- **recall multiplication and division facts for multiplication tables up to 12 × 12**
- **use place value, known and derived facts to multiply and divide mentally, including:**
  - **multiplying by 0 and 1;**
  - **dividing by 1;**
  - **multiplying together three numbers**
- **recognise and use factor pairs and commutativity in mental calculations**
- practise mental methods and extend this to three-digit numbers to derive facts, (for example 600 ÷ 3 = 200 can be derived from 2 x 3 = 6)
- apply understanding of the equals sign
- link facts within the tables (e.g. 5× is half of 10×)

Using the **distributive** law:
39 x 7 = 30 x 7 + 9 x 7
Using the **associative** law:
(2 x 3) x 4 = 2 x (3 x 4)

Using facts and rules:
2 x 6 x 5 = 10 x 6 = 60

Written Calculations

- **multiply two-digit and three-digit numbers by a one-digit number using formal written layout**
- Estimate before calculating
- Ensure written methods build on/relate to mental methods (e.g. grid method) based on an understanding of place value
- Use grid and expanded column methods as stepping stones alongside



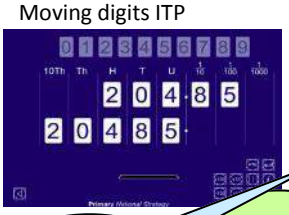
Key skills to support:
- know or quickly recall multiplication facts up to 12 × 12
- understand the effect of multiplying numbers by 10, 100 or 1000
- multiply multiples of 10, for example, 20 × 40;
- approximate, e.g. recognise that 72 × 38 is approximately 70 × 40 = 2800 and use this information to check whether their answer appears sensible

Revert to expanded methods if children find formal calculation method difficult

Representations to support mental and written calculations.

Ensure children can confidently multiply & divide by 10 and 100, that multiplying by 10 makes the number bigger and all digits move one place to the left, while dividing by 10 makes the number smaller and all the digits move one place to the right.

Moving digits ITP

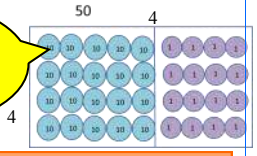


245
X 6
1470
23

This digit is worth 200

This digit is worth 30

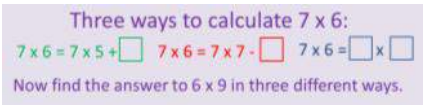
I can use place value counters to model the grid method



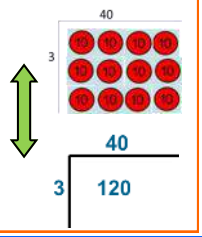
Use intelligent practice e.g.

2 × 3 =	6 × 7 =	9 × 8 =
2 × 30 =	6 × 70 =	9 × 80 =
2 × 300 =	6 × 700 =	9 × 800 =
20 × 3 =	60 × 7 =	90 × 8 =
200 × 3 =	600 × 7 =	900 × 8 =

Three ways to calculate 7 x 6:

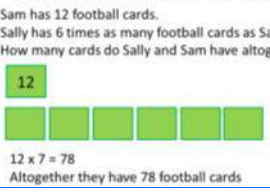


Use arrays made with place value counters to demonstrate the link between multiplication and division. This will support understanding of the grid method.



Children need to understand and apply the language of multiples and factors and use it in solving multiplication and division problems e.g.
'All factors of 36 are multiples of 2, true or false?
Find me two factors of 48 that are also multiples of 3.'

Using the bar model to solve problems:

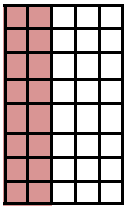


Fractions

- **recognise and show, using diagrams, families of common equivalent fractions**
- understand the relation between non-unit fractions and multiplication and division of quantities, with particular emphasis on tenths and hundredths.
- make connections between fractions of a length, of a shape and as a representation of one whole or set of quantities.
- use factors and multiples to recognise equivalent fractions and simplify where appropriate

$\frac{4}{10}$ $\frac{6}{15}$ $\frac{8}{20}$ $\frac{10}{25}$ $\frac{12}{30}$ $\frac{14}{35}$ $\frac{16}{40}$

$\frac{2}{5} = \frac{16}{40}$



Links from other strands

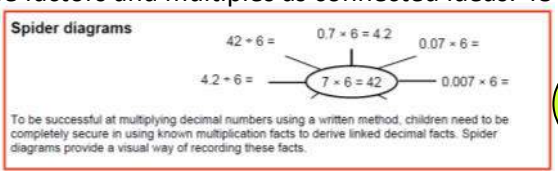
- **solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.**
- **Convert between different units of measure (e.g. km to m)** - use multiplication to convert from larger to smaller units
- *Understand the relation between non-unit fractions and multiplication/division of quantities. With particular emphasis on tenths and hundredths*
- *relate area to arrays and multiplication.*
- Problem solving work can involve finding all possibilities and combinations drawing on knowledge of multiplication tables facts
- *Pupils understand and use a greater range of scales in their representations* (Statistics)

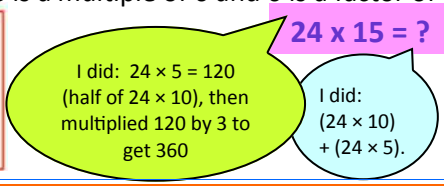
The Bridge Federation Calculation Policy for multiplication: Year 5

Informal methods to support mental Calculations

- **multiply and divide numbers mentally drawing upon known facts**
- **multiply and divide whole numbers and those involving decimals by 10, 100 & 1000**
- **Recognise and use square & cube numbers (& notation)**
- Use factors and multiples as connected ideas: 48 is a multiple of 6 and 6 is a factor of 48.

Pupils should be taught throughout that percentages, decimals and fractions are different ways of expressing proportions.

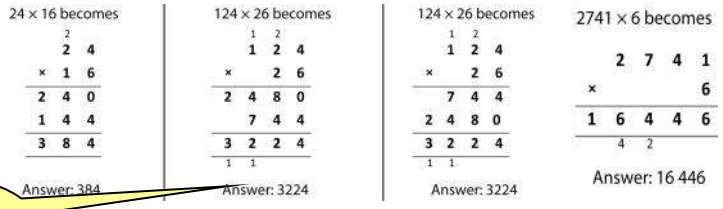




Example of constructing equivalence statements:
4 x 35 = 2 x 2 x 35;
3 x 270 = 3 x 3 x 9 x 10
= 92 x 10

Written Calculations

- **multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers**

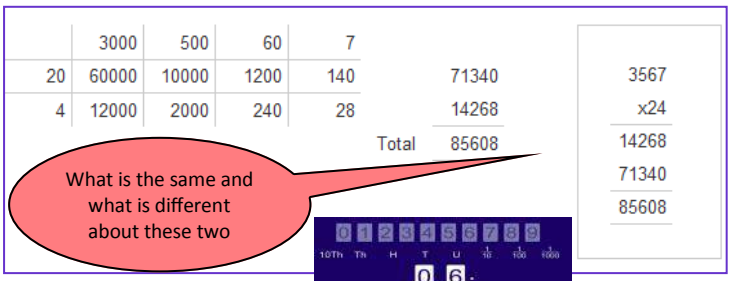


Compact methods for multiplication are efficient but often do not make the value of each digit explicit. When introducing multiplication of decimals, it is sensible to take children back to an expanded form such as the grid method where the value of each digit is clear, to ensure that children understand the process.

Does your answer seem reasonable?

Revert to expanded methods if children find formal calculation method difficult (see Y3/Y4)

Representations to support mental and written calculations.

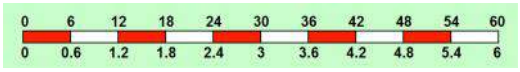


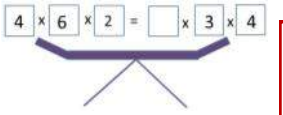
	3000	500	60	7	
20	60000	10000	1200	140	71340
4	12000	2000	240	28	14268
				Total	85608

3567
x24
14268
71340
85608

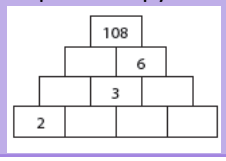
What is the same and what is different about these two

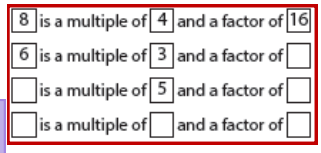
12 x 6 = 24 x 3
True or false? Prove it!



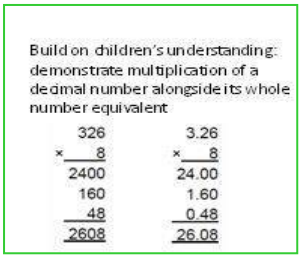


Complete the pyramid:





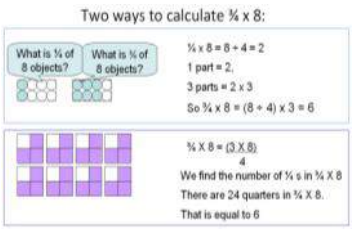
Start multiplying by using the **least significant digit** for the grid method will support children with implementation of the written procedure



Fractions

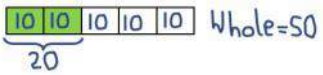
- **multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams**
- **identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths**

Pupils connect multiplication by a fraction to using fractions as operators (fractions of), and to division, building on work from previous years. This relates to scaling by simple fractions, including fractions > 1.



Encourage children to draw diagrams to represent situations or problems involving fractions.
Model how to do this, for example:

$^2/_5$ of a number is 20. What is the number?



Links from other strands

- **identify multiples & factors, including finding all factor pairs of a number, & common factors of two numbers**
- **know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers**
- **establish whether a number up to 100 is prime and recall prime numbers up to 19**
- **solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes, and including understanding the meaning of the equals sign**
- **solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates**
- **use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation, including scaling.**
- **convert between different units of metric measure; problems including money,.**

Other links: ratio,

Pupils use their knowledge of place value and multiplication and division to convert between standard units.

Pupils calculate the perimeter of rectangles and related composite shapes, including using the relations of perimeter or area to find unknown lengths. Missing measures questions such as these can be expressed algebraically, for example 4 + 2b = 20 for a rectangle of sides 2 cm and b cm and perimeter of 20cm.

Pupils calculate the area from scale drawings using given measurements.

The Bridge Federation Calculation Policy for multiplication: Year 6

Informal methods to support mental Calculations

- **perform mental calculations, including with mixed operations and large numbers** *(increasingly large numbers & more complex calculations)*
- *use all the multiplication tables to calculate mathematical statements in order to maintain fluency.*
- **use estimation to check answers to calculations & determine, in the context of a problem, an appropriate degree of accuracy.**
- **identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places**
- Use and apply connections between factors, multiples and prime numbers and between fractions, division and ratios.

Use mental strategies to solve problems e.g.
- x4 by doubling and doubling again
- x5 by x10 and halving
- x20 by x10 and doubling
- x9 by multiplying by 10 and adjusting
- x6 by multiplying by 3 and doubling

What is the best approximation for 4.4 x 18.6?

Children should know the square numbers up to 12 × 12 & derive the corresponding squares of multiples of 10 e.g. 80 × 80 = 6400

How many different x/÷ facts can you make using 72? 7.2? 0.72?

Written Calculations

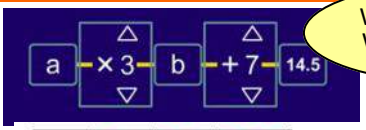
- **multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication** *(short & long multiplication)*
- **multiply one-digit numbers with up to two decimal places by whole numbers**
- understand that standard written multiplication method involves a number of partial products e.g. 36 × 24 is made up of four partial products 30 × 20, 30 × 4, 6 × 20, 6 × 4.
- Use manipulatives to support structure of the algorithm especially place value

£	6.23
x	27
	43.61 (1 2)
	124.60
£	168.21 (1)

Revert to expanded methods if children find formal calculation method difficult (see Y4/Y5)

Representations to support mental and written calculations.

Look at long-multiplication calculations containing errors, identify the errors and determine how they should be corrected



What's the same? What's different?

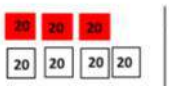
x	8	0.4	0.06
11	88	4.4	0.66

= 93.06

8.46 X 11 = 93.06

Using the bar model to solve problems:

A gardener plants tulip bulbs in a flower bed. She plats 3 red bulbs for every 4 white bulbs. She plants 60 red bulbs. How many white bulbs does she plant?



Use questioning to develop conceptual understanding e.g.
Which is the odd one out?
24 x 3 36 x 4 13 x 5 32 x 2

Use empty box questions:

□ × □ = 864

□ × □ × □ = 864

BODMAS
- 8·4 × 3 + 8·4 × 7
- 6·7 × 5 − 0·67 × 50
- 93 × 0·2 + 0·8 × 93
- 7·2 × 4 + 3·6 × 8

Fractions

- **multiply simple pairs of proper fractions, writing the answer in its simplest form e.g. ¼ x ½ = 1/8**

Three key applications of understanding:
- Recognise that ¼ of 12, ¼ x 12 and 12 divided by 4 are equivalent
- Use cancellation to simplify the product of a fraction and an integer e.g. ⅓ x 15 = 3, ⅔ x 15 = 2 x ⅓ x 15 = 2x3 = 6
- Work out how many ⅓s in 15, how many ⅔s in 15, how many 2/5s in 1 etc.

To calculate ¼ x ½ , find ½ of a rectangle/array, then divide that ½ into ¼s. So ¼ of ½ is 1/8

Pupils should use a variety of images to support their understanding of multiplication with fractions. This follows earlier work about fractions as operators (fractions of), as numbers, and as equal parts of objects, e.g. as parts of a rectangle.

Links from other strands

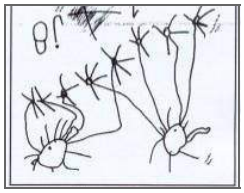
- **identify common factors, common multiples and prime numbers**
- **use their knowledge of the order of operations to carry out calculations involving the four operations**
- **solve problems involving addition, subtraction, multiplication and division**
- *explore the order of operations using brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9.*
- **Fractions, decimals and percentages** including equivalences in different contexts.
- **solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts**
- **solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison**
- **solve problems involving similar shapes where the scale factor is known or can be found**
- **solve problems involving unequal sharing and grouping using knowledge of fractions and multiples.**
- **Algebra** including formulae, linear number sequences, combinations of variables
- **Measurement** including solving problems with conversion of units, decimal notation, area & volume
- **Statistics** including pie charts, line charts and calculating the mean

The Bridge Federation Calculation Policy for division: Year 1

Mental Calculations

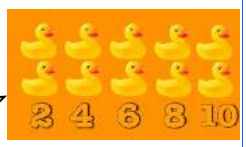
Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

(Pupils) make connections between arrays, number patterns, and counting in twos, fives and tens.



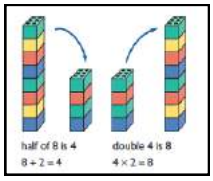
Count on or back in 2s, 5s and 10s and look for patterns.

Songs are useful for counting in steps.

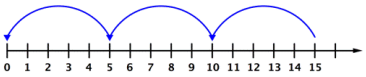


Written Calculations

Pictorial jottings to support the calculation of 8 ÷ 4



Children should experiment with the concepts of sharing and grouping in a number of contexts. Initially they use their own recording—moving towards fluent, symbolic notation in Year 2. Conceptual understanding and recording should be continuously supported by the use of **arrays** as a default model, as well as other representations, (see below.)

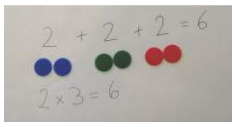


The relationship between multiplication and division must be continually considered.

Representations to support mental and written calculations.

Use a range of concrete and pictorial representations, including:

- Manipulatives to support children's own recording; and understanding of *sharing* and the link with multiplication.
  "*How can we share 6 cakes between 2 people?*"



Here, the cakes are placed in an array formation.

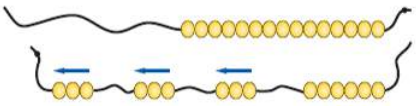
Moving from concrete to pictorial, counters represent the cakes to reinforce the relationship between multiplication and division.

How many 2 tiles can we fit on the 6 tile?

- Manipulatives, and real-life objects to support children's own recording; and understanding of *grouping* and the link with multiplication.

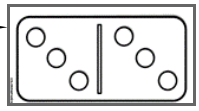
Coat hangers and socks support calculation of 8÷2

Bead strings



15 ÷ 2 using grouping model

"Double 3 is 6. Half of 6 is 3."



- Dominoes and dice to reinforce concepts of doubling and halving.

Fractions

Recognise, find and name a half as one of two equal parts of an object, shape or quantity
Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity. (*See* Representations *above.)*

Links from other strands

They practise counting as reciting numbers and counting as enumerating objects, and counting in twos, fives and tens from different multiples to develop their recognition of patterns in the number system (for example, odd and even numbers). (PLACE VALUE).
Pupils are taught half and quarter as 'fractions of' by solving problems using shapes, objects and quantities. (FRACTIONS)

The Bridge Federation Calculation Policy for division: Year 2

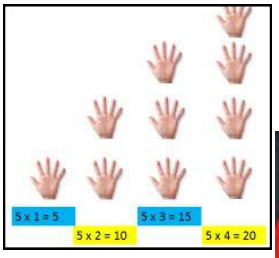
Division and multiplication concepts must be linked continuously.

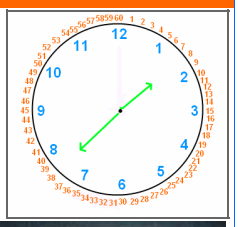
Mental Calculations

The relationship between multiplication and division must be continually considered.

- **Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers .**
- **Calculate mathematical statements for multiplication and division within**
- **the multiplication tables and write them using the**

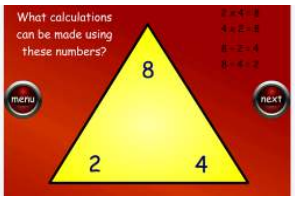
"5, one time", "5, two times" and so on.



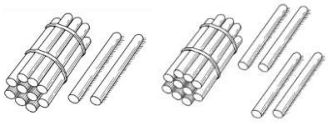


Written Calculations

- **Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot**
- **Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts. (*See below.*)**



"There are 26 straws. ½ of the straws is equal to 13 straws."



½ of 26 = 13
26 ÷ 2 = 13

Pupils decode a problem first, represent it using manipulatives and jottings; and finally record it symbolically.

Representations to support mental and written calculations.

Use a range of concrete and pictorial representations, including:

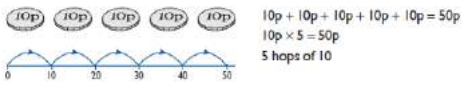
- Arrays

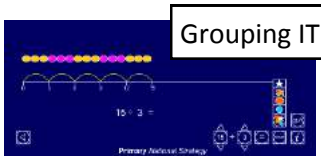
7 x 2 = 14
14 ÷ 2 = 7

2 x 7 = 14
14 ÷ 7 = 2

Is 14 an odd number? How do you know?

- Number lines to support grouping





"How many groups of 5 minutes have passed when the minute hand reaches twenty past?"

- Representations to support multiplicative reasoning:

Using Dienes: *"If 40 ÷ 10 = 4 and 30 ÷ 10 = 3, what do you think 70 ÷ 10 would be? Why?"*

Fractions

Recognise, find, name and write fractions ⅓, ¼, ¾, 2/4 of a length, shape, set of objects or quantity
Write simple fractions for example, ½ of 6 = 3 and recognise the equivalence of ½ and 2/4.

Links from other strands

- **Count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward.**
- **Recognise the place value of each digit in a two-digit number (tens, ones) (PLACE VALUE).**
- **Tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times, (MEASURES).**

The Bridge Federation Calculation Policy for division: Year 3

Mental Calculations

Pupils should be taught to recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.

Pupils continue to practise their mental recall of multiplication tables... in order to improve fluency. Pupils develop efficient mental methods, for example, using commutativity and associativity (e.g., 4 × 12 × 5 = 4 × 5 × 12 = 20 × 12 = 240) and multiplication and division facts to derive related facts.

36 ÷3 = 12

30 6

30 ÷3=10 6 ÷3=2

+

"4 x 3 is 12, so 12 ÷ 3 = 4."

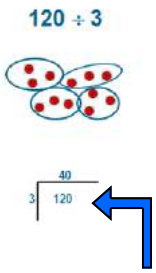
Written Calculations

Pupils should be taught to:

•write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.

•solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects, (see Links from other strands, below.)

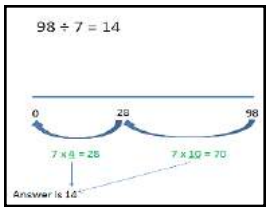
"I know 6÷3=2, so 60÷3=20."
"I know 12÷3=4, so 120÷3=40."



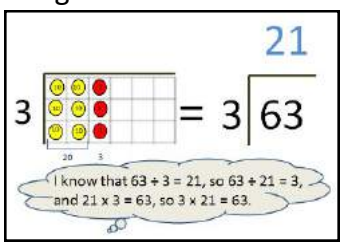
New written methods can be modelled alongside mental or informal methods to ensure understanding.

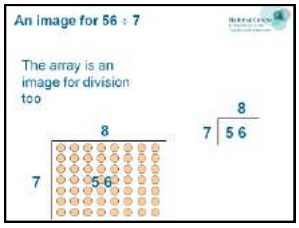
Representations to support mental and written calculations.

Use a range of concrete and pictorial resources, including:



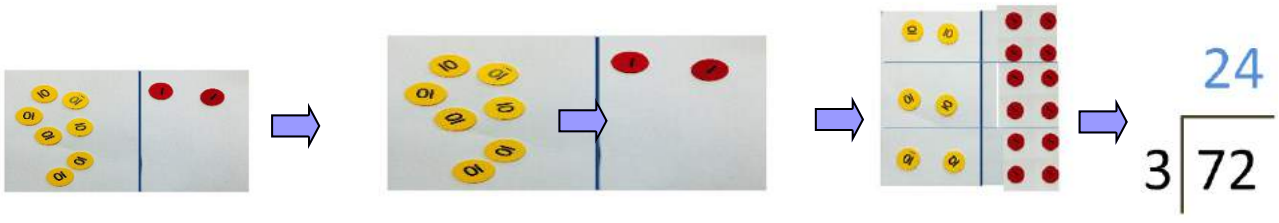
63 ÷ 3 equals three groups of 2 tens and a one.





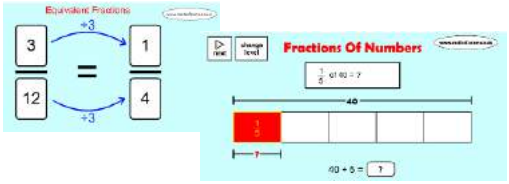
How could I calculate 72÷3 ?

Informal exploration with manipulatives supports the progression to formal written methods—which is continued in Year 4.

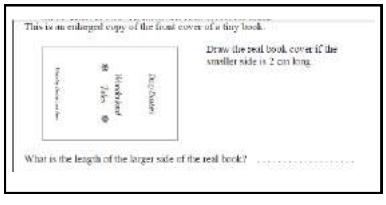


Fractions

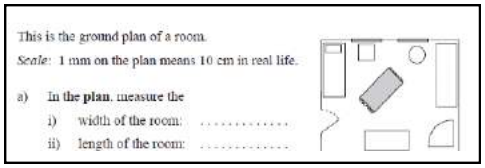
- **Recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10.**
- **Recognise and show, using diagrams, equivalent fractions with small denominators.**
- **Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators.**



Links from other strands



Pupils solve simple problems in contexts, including measuring and scaling contexts, (e.g., four times as high etc.) and correspondence problems.



The Bridge Federation Calculation Policy for division: Year 4

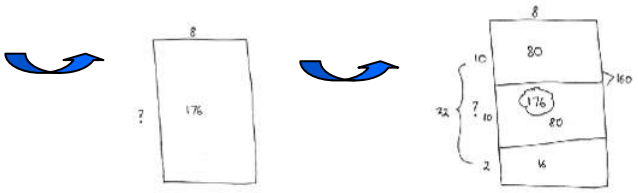
Informal methods to support mental Calculations

Pupils should be taught to:

- **recall multiplication and division facts for multiplication tables up to 12 × 12**
- **use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers**
- **recognise and use factor pairs and commutativity in mental calculations**

I know that 6÷3=2, so 600÷3=2.

Using known facts and blank arrays to calculate 176÷8.



176÷8 = 22

Pupils practise mental methods and extend this to three-digit numbers to derive facts.

Written Calculations

Pupils should be taught to:

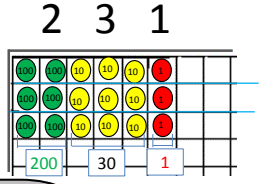
- **multiply two-digit and three-digit numbers by a one-digit number using formal written layout**
- **solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.**

Pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers .

Revert to expanded methods if children find formal calculation method difficult

Representations to support mental and written calculations.

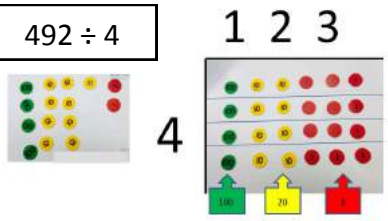
693 ÷ 3



Children can work in pairs: child A constructs the array (dividing manipulatives into 3 rows), child B checks it and records this in a formal, short division format.

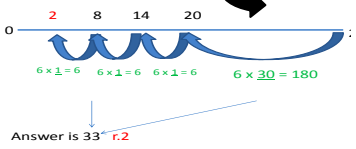
By working through larger number calculations with manipulatives, children gain experience of ex-change (re-partitioning) within division algorithms.

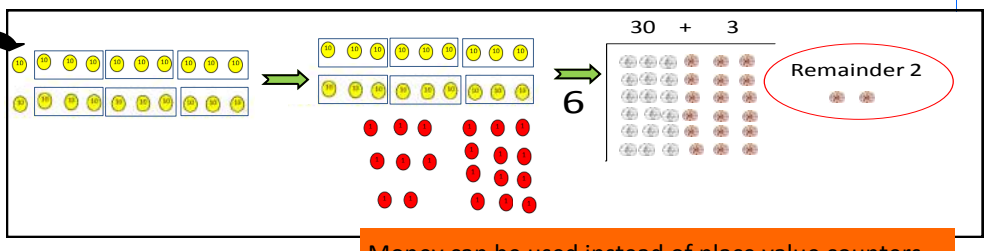
492 ÷ 4



By the end of Year 4, children need to have encountered remainders in a number of contexts. Pupils can be introduced to remainders using known facts: e.g. 13÷4; and then progress to larger numbers. (See below).

200 ÷ 6 = 33 r.2





Money can be used instead of place value counters.

Fractions

Pupils should be taught to:

- ***recognise and show, using diagrams, families of common equivalent fractions***
- ***recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten.***
- ***solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number***
- ***find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths***

Links from other strands

- **Convert between different units of measure [for example, kilometre to metre; hour to minute]**
- **Estimate, compare and calculate different measures, including money in pounds and pence (MEASURES)**
- **Recognise that hundreths arise when dividing an object by one hundred and dividing tenths by ten. (FRACTIONS)**

The Bridge Federation Calculation Policy for division: Year 5

Informal methods to support mental Calculations

. Pupils should be taught to:

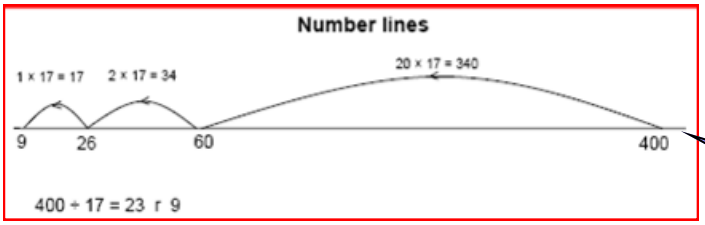
- **multiply and divide whole numbers and those involving decimals by 10, 100 and 1000**
- **multiply and divide numbers mentally drawing upon known facts**

identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers .

If 42 ÷ 6 = 7

÷ 10 ÷ 10

Then 4.2 ÷ 6 = 0.7



Factorising

480 ÷ 15

= 480 ÷ 5 ÷ 3

"I know that the answer to 138 ÷ 6 will be close to 20, because 2 x 6 = 12, so 20 x 6 = 120."

Pupils apply all the multiplication tables and related division facts frequently and use them confidently .

Written Calculations

Pupils practise and extend their use of the formal written methods of short multiplication and short division.

- **Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.**

98 ÷ 7 becomes

7 | 9 (2) 8 → 1 4

Answer: 14

432 ÷ 5 becomes

5 | 4 3 (3) 2 → 8 6 r2

Answer: 86 remainder 2

496 ÷ 11 becomes

11 | 4 9 (5) 6 → 4 5 r1

Answer: 45 1/11

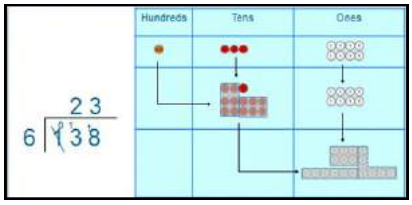
- **Pupils interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding. (See Representations below.)**

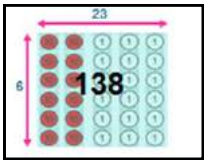
Revert to expanded methods if children find formal calculation method difficult

Representations to support mental and written calculations.

Can we divide this 100 token into 6 equal groups? , then we must exchange it for ten 10 tokens. Can we divide into 6 groups now?

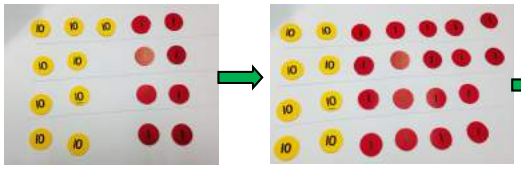
Short division with exchange.

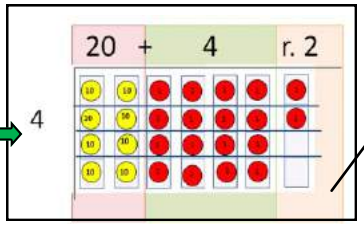




Practical experience with manipulatives is vital for children to talk through the language of division e.g. *exchange, remainder*; and to embed conceptual understanding.

Understanding remainders.





2 out of a whole group of 4 = 2/4 = 1/2 = 0.5

98 ÷ 4 = 98/4 = 24 r 2 = 24 1/2 = 24.5

What is the same? What's different about the ways that these remainders are expressed?

Fractions

- **Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number .**
- ***Pupils connect equivalent fractions > 1 that simplify to integers with division and other fractions > 1 to division with remainders.***
- ***Pupils connect multiplication by a fraction to using fractions as operators (fractions of), and to division.***
- ***Pupils should make connections between percentages, fractions and decimals***

Links from other strands

- ***Pupils use all four operations in problems involving time and money, including conversions. …..using decimal notation, including scaling.***
- ***calculate and compare the area of rectangles (including squares). (MEASURES)***

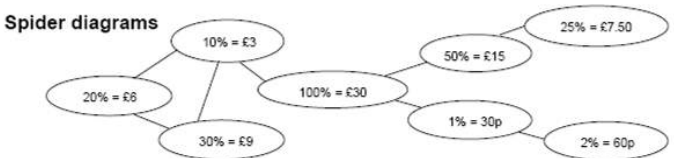
- **establish whether a number up to 100 is prime and recall prime numbers up to 19**
- **recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3)**
- **solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes and including scaling by simple fractions and problems involving simple rates.**
- **solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign. (NUMBER—MULTIPLICATION AND DIVISION)**

The Bridge Federation Calculation Policy for division: Year 6

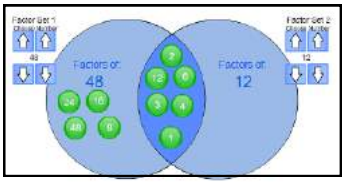
Informal methods to support mental Calculations

Pupils should be taught to:

- perform mental calculations, including with mixed operations and large numbers.
- use their knowledge of the order of operations to carry out calculations involving the four operations.
- identify common factors, common multiples and prime numbers.



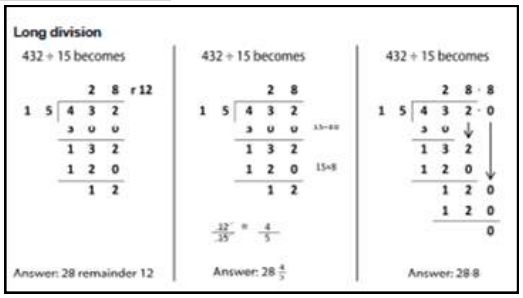
I know that 366 will divide by 6 because it has 2 and 3 as factors



- Solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

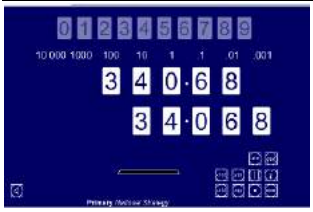
Written Calculations

- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.
- *Pupils practise division for larger numbers, using the formal written methods of short and long division.*

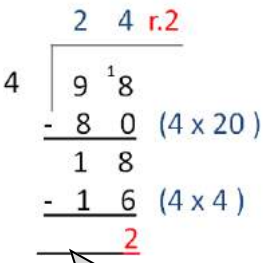


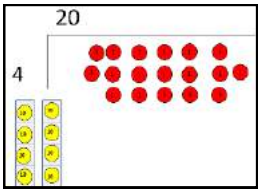
Revert to expanded methods if children find formal calculation method difficult

Representations to support mental and written calculations.



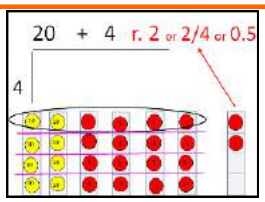
To introduce the long division model, use a calculation which can be represented both with manipulatives and by a short division algorithm. Use questioning and discussion to compare written methods.



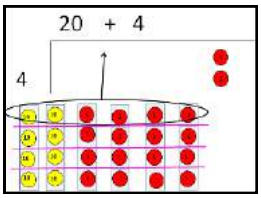


£1362.72 ÷ 40 = ?

£1362.72 ÷ 4 = £340.68 [½ and ½ again.] £340.68 ÷ 10 = £34.068 which rounds to £34.07.

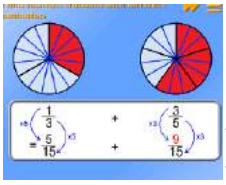


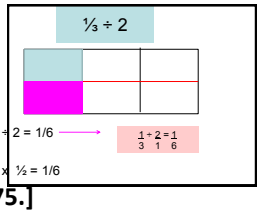
What's the same? What's different?



Fractions

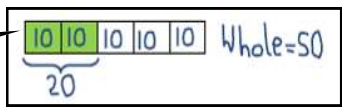
- use common factors to simplify fractions,
- compare and order fractions, including fractions > 1
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- divide proper fractions by whole numbers [for example, 1/3 ÷2 = 1/6 .]
- associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375.]





- *Pupils use their understanding of the relationship between unit fractions and division to work backwards.* use written division methods in cases where the answer has up to 2 dp.

2/5 of a number is 20. What is the number?



Links from other strands

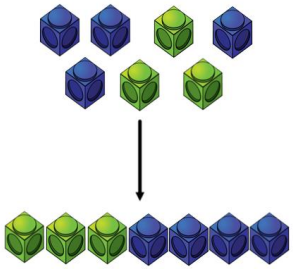
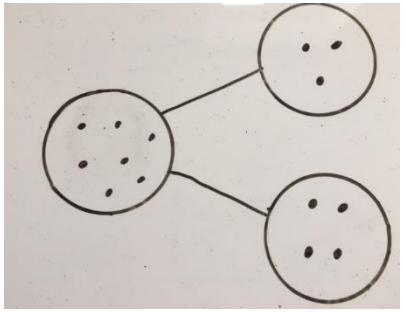
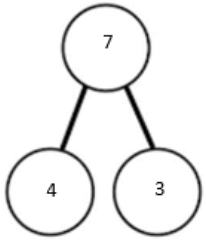
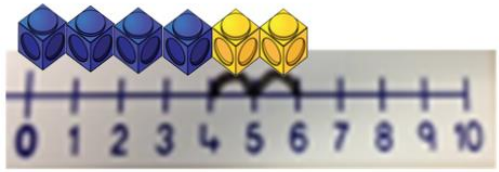
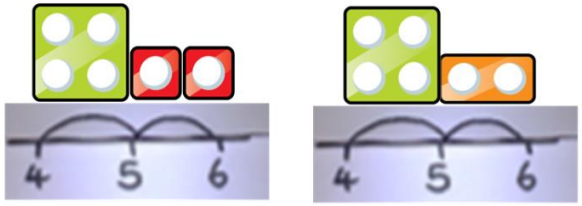
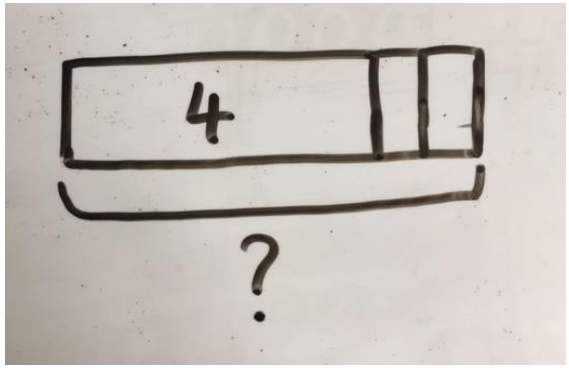
- *Pupils are introduced to the division of decimal numbers by one-digit whole number, initially, in practical contexts involving measures and money. They recognise division as the inverse of multiplication.*
- *Pupils also develop their skills of rounding and estimating. This includes rounding answers to a specified degree of accuracy and checking the reasonableness of their answers.* (FRACTIONS)

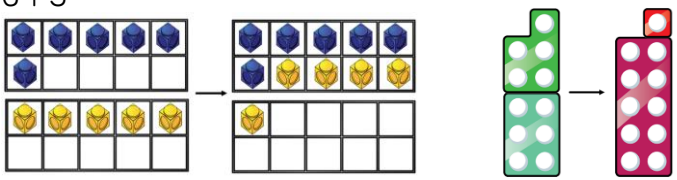
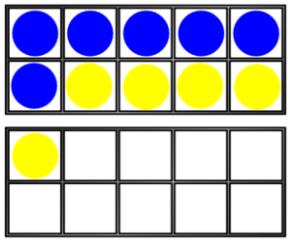
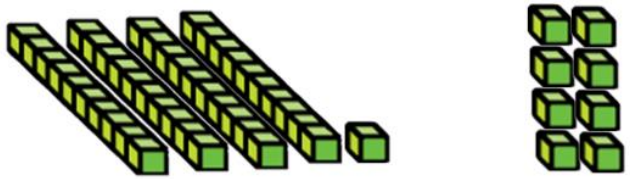
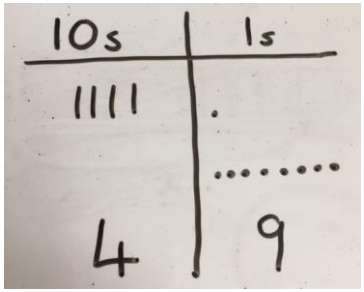
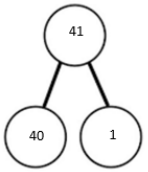
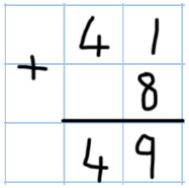
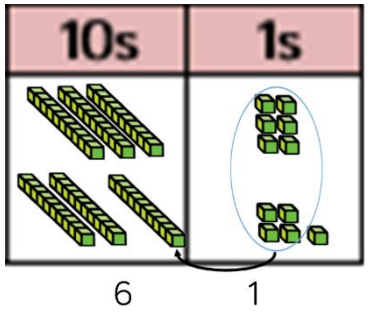
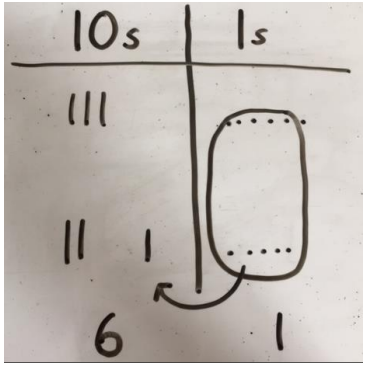
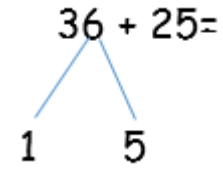
"8 is the best estimate for 72.34 ÷ 8.91; because the numbers in the algorithm can be rounded to 72 ÷ 9."

- *solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate.*
- *use, read, write and convert between standard units….using decimal notation to up to 3d.p. (MEASURES)*
- *interpret and construct pie charts and line graphs and use these to solve problems*
- *calculate and interpret the mean as an average. (STATISTICS)*
- *solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts (RATIO AND PROPORTION)*

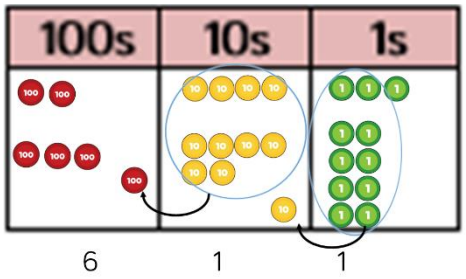
CPA approach : Addition

Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

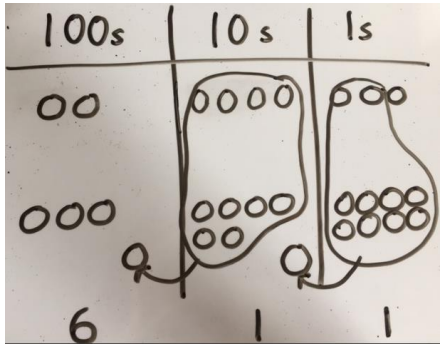
Concrete	Pictorial	Abstract
Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars). 	Children to represent the cubes using dots or crosses. They could put each part on a part whole model too. 	4 + 3 = 7 Four is a part, 3 is a part and the whole is seven. 
Counting on using number lines using cubes or Numicon.  	A bar model which encourages the children to count on, rather than count all. 	The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4 + 2 

Regrouping to make 10; using ten frames and counters/cubes or using Numicon. 6 + 5 	Children to draw the ten frame and counters/cubes. 	Children to develop an understanding of equality e.g. $6 + \square = 11$ $6 + 5 = 5 + \square$ $6 + 5 = \square + 4$
TO + O using base 10. Continue to develop understanding of partitioning and place value. 41 + 8 	Children to represent the base 10 e.g. lines for tens and dot/crosses for ones. 	41 + 8  1 + 8 = 9 40 + 9 = 49 
TO + TO using base 10. Continue to develop understanding of partitioning and place value. 36 + 25 	Chidlren to represent the base 10 in a place value chart. 	Looking for ways to make 10.  30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 Formal method: 36 +25 61 1

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.

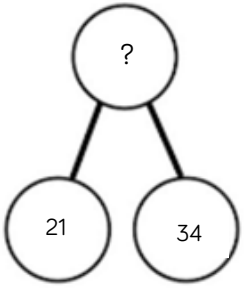


Chidren to represent the counters in a place value chart, circling when they make an exchange.



$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 1\ 1 \end{array}$$

Conceptual variation; different ways to ask children to solve 21 + 34



?	
21	34

Word problems:
In year 3, there are 21 children and in year 4, there are 34 children.
How many children in total?

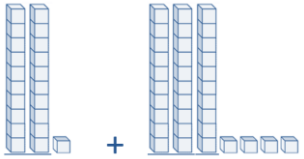
21 + 34 = 55. Prove it

$$\begin{array}{r} 21 \\ +34 \\ \hline \\ \hline \end{array}$$

21 + 34 =

$\square$ = 21 + 34

Calculate the sum of twenty-one and thirty-four.

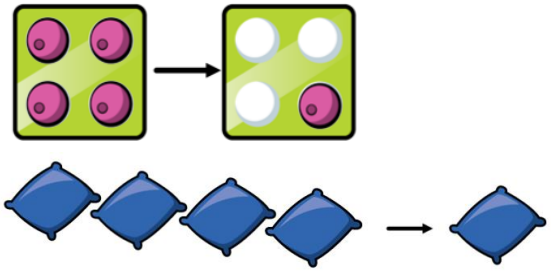
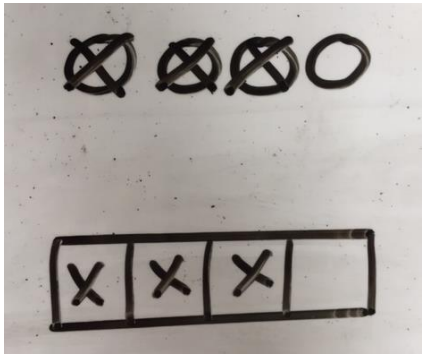
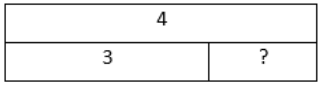
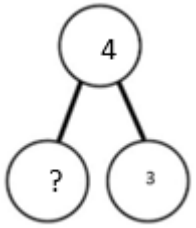
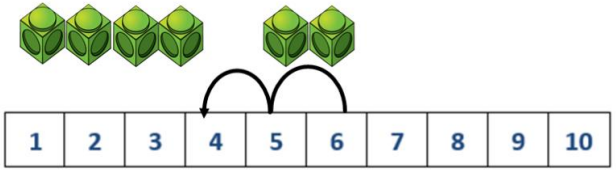
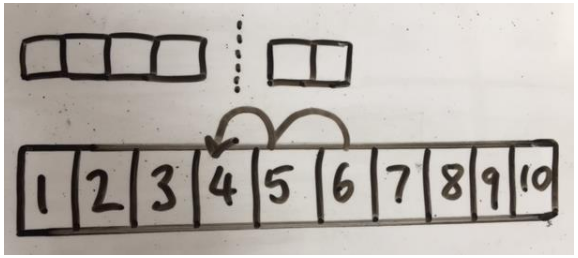
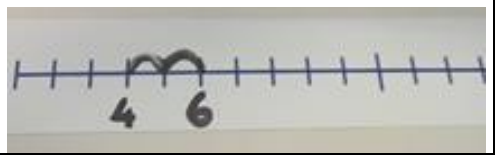


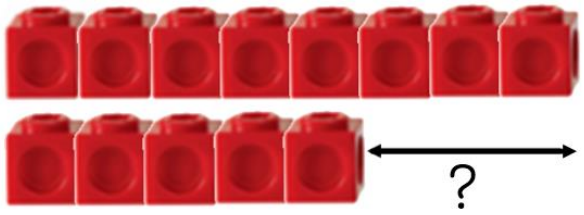
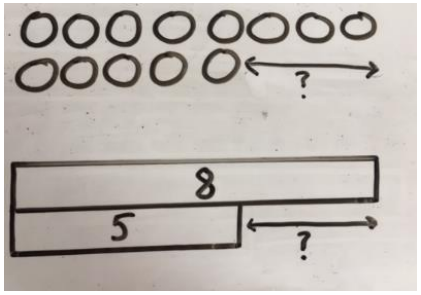
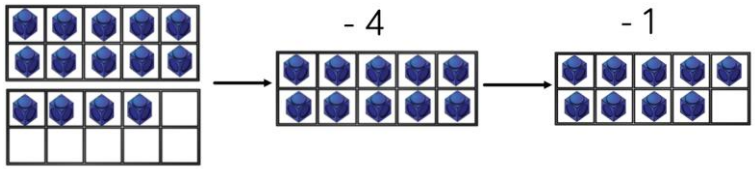
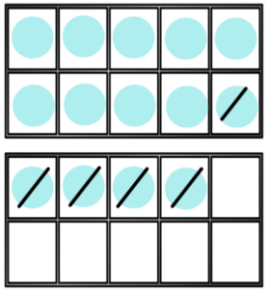
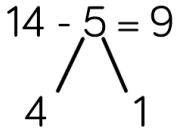
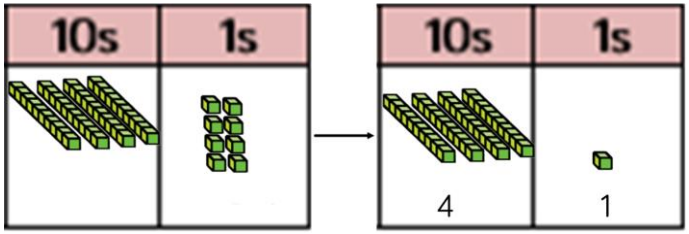
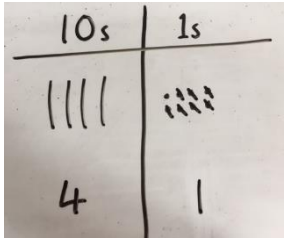
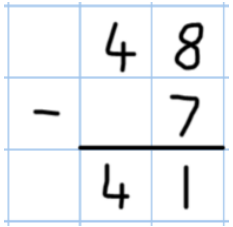
Missing digit problems:

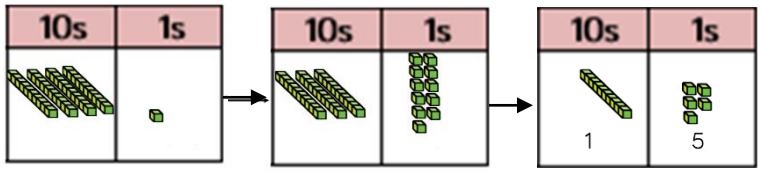
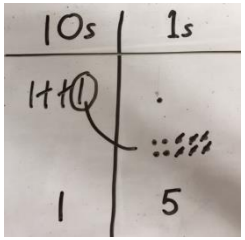
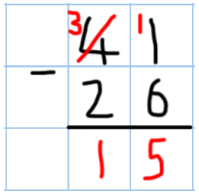
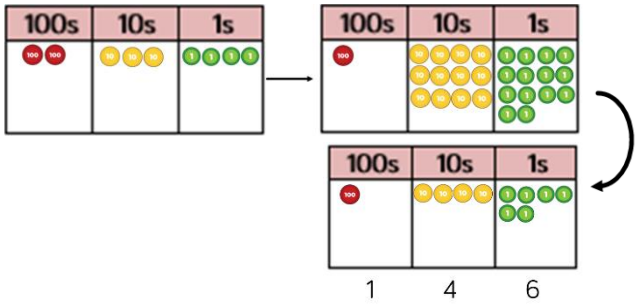
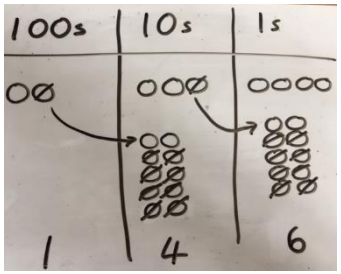
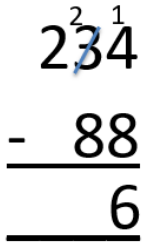
10s	1s
10 10	1
10 10 10	?
?	5

CPA Approach : Subtraction

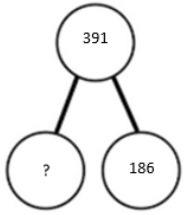
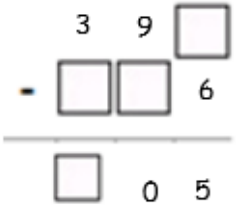
Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used). 4 – 3 = 1 	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. 	4- 3 = □ = 4 – 3  
Counting back (using number lines or number tracks) children start with 6 and count back 2. 6 – 2 = 4  	Children to represent what they see pictorially e.g.  	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line  

Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used). Calculate the difference between 8 and 5. 	Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate. 	Find the difference between 8 and 5. 8 – 5, the difference is ☐ Children to explore why 9 - 6 = 8 – 5 = 7 – 4 have the same difference.
Making 10 using ten frames. 14 – 5 	Children to present the ten frame pictorially and discuss what they did to make 10. 	Children to show how they can make 10 by partitioning the subtrahend.  14 – 4 = 10 10 – 1 = 9
Column method using base 10. 48-7 	Children to represent the base 10 pictorially. 	Column method or children could count back 7. 

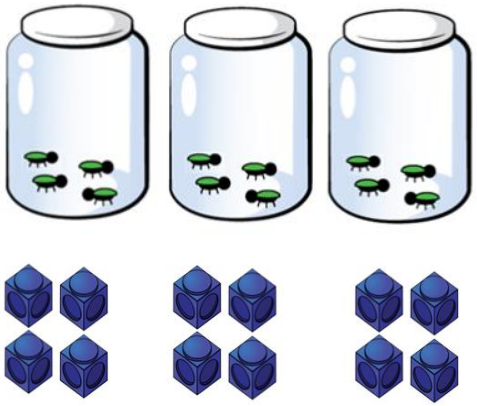
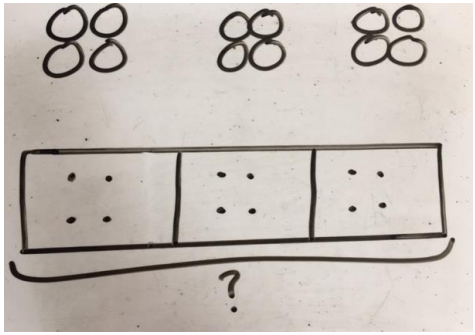
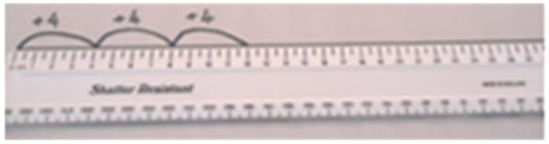
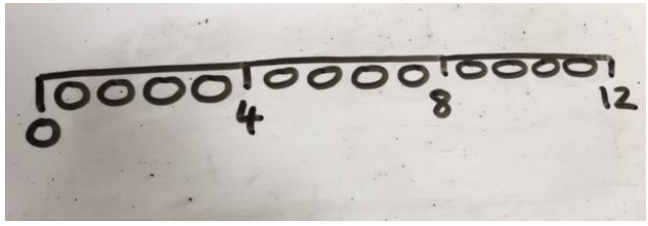
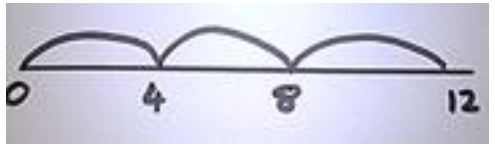
Column method using base 10 and having to exchange. 41 – 26 	Represent the base 10 pictorially, remembering to show the exchange. 	Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11. 
Column method using place value counters. 234 – 88 	Represent the place value counters pictorially; remembering to show what has been exchanged. 	Formal colum method. Children must understand what has happened when they have crossed out digits. 

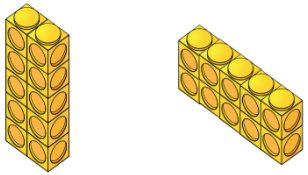
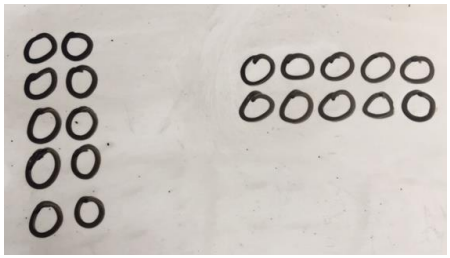
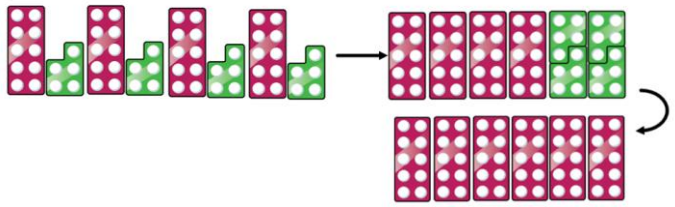
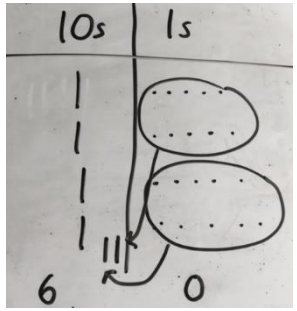
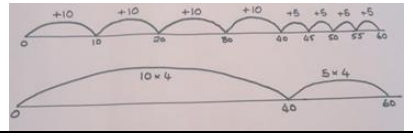
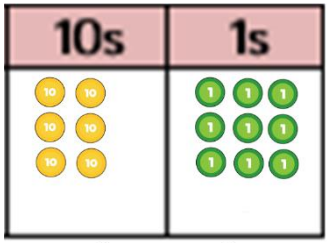
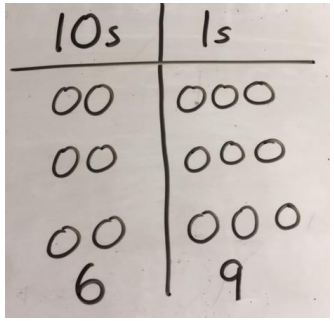
Conceptual variation; different ways to ask children to solve 391 - 186

 391 186 ?	Raj spent £391, Timmy spent £186. How much more did Raj spend? Calculate the difference between 391 and 186.	☐ = 391 – 186 391 -186 ___ What is 186 less than 391?	Missing digit calculations 

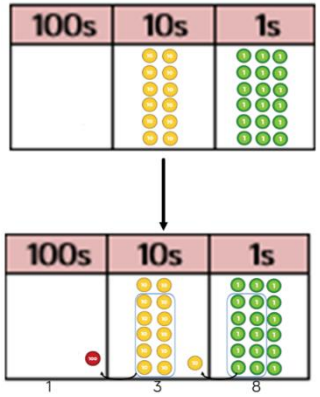
CPA approach : Multiplication

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

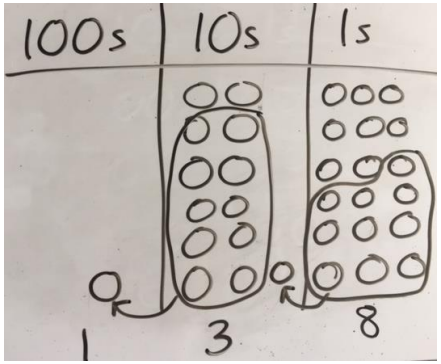
Concrete	Pictorial	Abstract
Repeated grouping/repeated addition 3 × 4 4 + 4 + 4 There are 3 equal groups, with 4 in each group. 	Children to represent the practical resources in a picture and use a bar model. 	3 × 4 = 12 4 + 4 + 4 = 12
Number lines to show repeated groups- 3 × 4   Cuisenaire rods can be used too.	Represent this pictorially alongside a number line e.g.: 	Abstract number line showing three jumps of four. 3 × 4 = 12 

Use arrays to illustrate commutativity counters and other objects can also be used. $2 \times 5 = 5 \times 2$  2 lots of 5 5 lots of 2	Children to represent the arrays pictorially. 	Children to be able to use an array to write a range of calculations e.g. $10 = 2 \times 5$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$
Partition to multiply using Numicon, base 10 or Cuisenaire rods. 4×15 	Children to represent the concrete manipulatives pictorially. 	Children to be encouraged to show the steps they have taken. 4 x 15 10 5 10 x 4 = 40 5 x 4 = 20 40 + 20 = 60 A number line can also be used 
Formal column method with place value counters (base 10 can also be used.) 3×23  6 9	Children to represent the counters pictorially. 	Children to record what it is they are doing to show understanding. 3×23 20 3 $3 \times 20 = 60$ $3 \times 3 = 9$ $60 + 9 = 69$ $\begin{array}{r} 23 \\ \times\ 3 \\ \hline 69 \end{array}$

Formal column method with place value counters.
6 x 23



Children to represent the counters/base 10, pictorially e.g. the image below.



Formal written method

$$6 \times 23 =$$

$$\begin{array}{r} 23 \\ \times\ 6 \\ \hline 138 \\ \hline 1\ 1 \end{array}$$

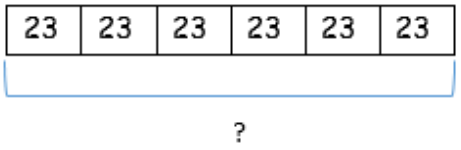
When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract:

To get 744 children have solved 6 × 124.
To get 2480 they have solved 20 × 124.

$$\begin{array}{r} 1\ 2\ 4 \\ \times\ \ \ 2\ 6 \\ \hline 7\ 4\ 4 \\ 2\ 4\ 8\ 0 \\ \hline 3\ 2\ 2\ 4 \\ \hline 1\ 1 \end{array}$$

Answer: 3224

Conceptual variation; different ways to ask children to solve 6 × 23



Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that 6 x 23 = 138

Find the product of 6 and 23

6 × 23 =

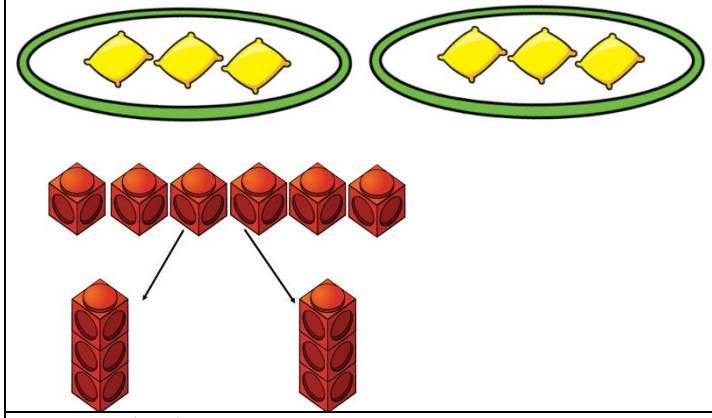
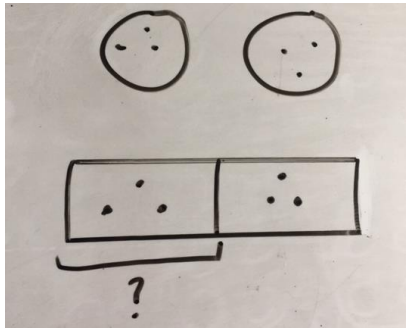
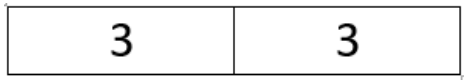
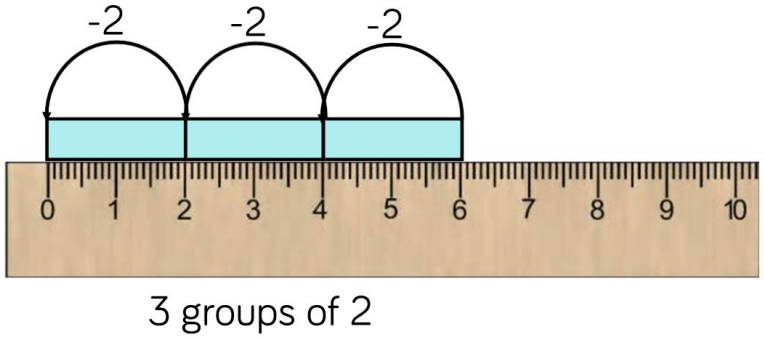
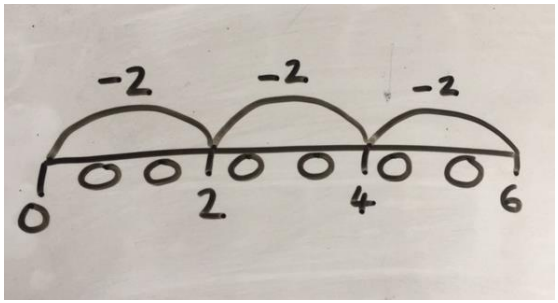
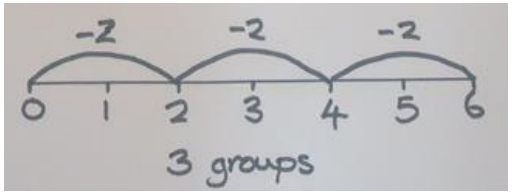
☐ = 6 × 23

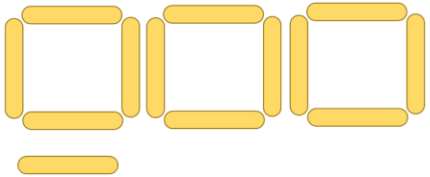
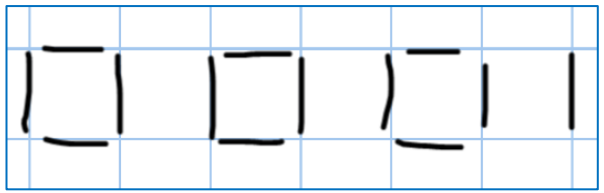
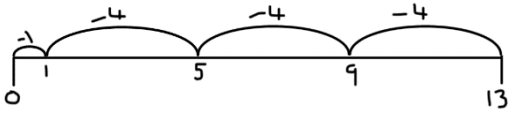
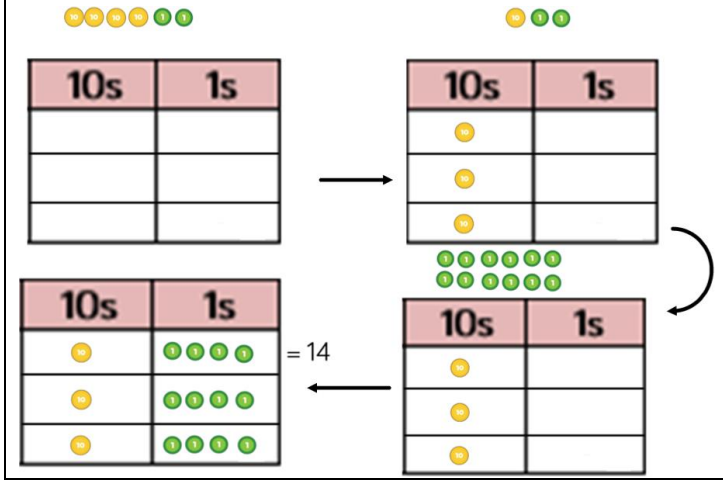
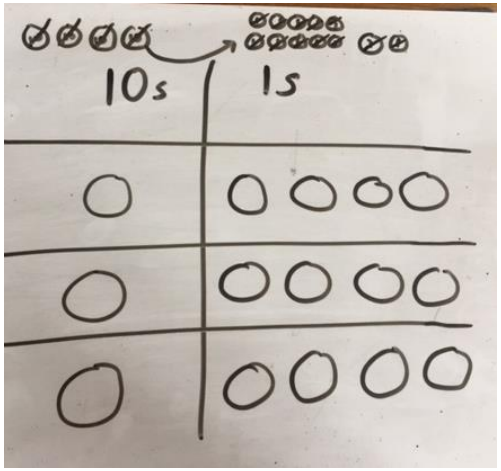
$$\begin{array}{r} 6 \\ \times\ 23 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 23 \\ \times\ 6 \\ \hline \\ \hline \end{array}$$

What is the calculation?
What is the product?

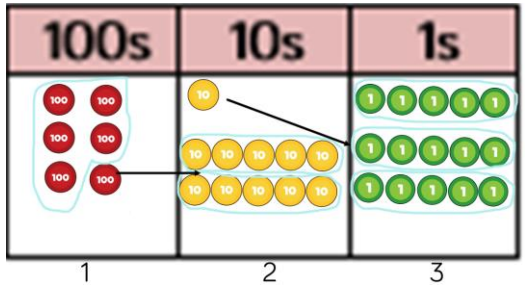
CPA approach : Division

Key language: share, group, divide, divided by, half.

Concrete	Pictorial	Abstract
Sharing using a range of objects. 6 ÷ 2 	Represent the sharing pictorially. 	6 ÷ 2 = 3  Children should also be encouraged to use their 2 times tables facts.
Repeated subtraction using Cuisenaire rods above a ruler. 6 ÷ 2  3 groups of 2	Children to represent repeated subtraction pictorially. 	Abstract number line to represent the equal groups that have been subtracted. 

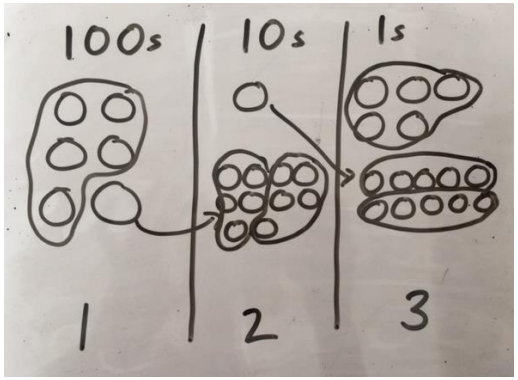
2d ÷ 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used. 13 ÷ 4 Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.  There are 3 whole squares, with 1 left over.	Children to represent the lollipop sticks pictorially.  There are 3 whole squares, with 1 left over.	13 ÷ 4 = 3 remainder 1 Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line. '3 groups of 4, with 1 left over' 
Sharing using place value counters. 42 ÷ 3 = 14 	Children to represent the place value counters pictorially. 	Children to be able to make sense of the place value counters and write calculations to show the process. 42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14

Short division using place value counters to group.
615 ÷ 5



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.

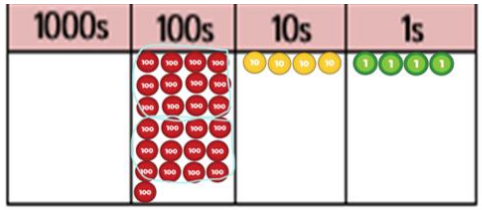


Children to the calculation using the short division scaffold.

$$5\overline{)615}$$
$$123$$
$$^{1}\,^{1}$$

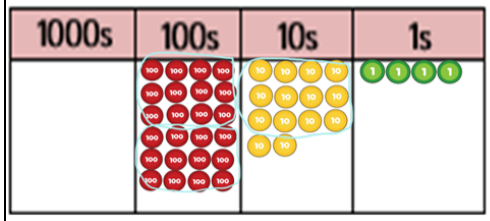
Long division using place value counters
2544 ÷ 12

We can't group 2 thousands into groups of 12 so will exchange them.



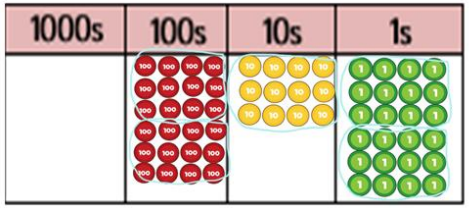
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$12\overline{)2544}$$
$$02$$
$$24$$
$$1$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$$

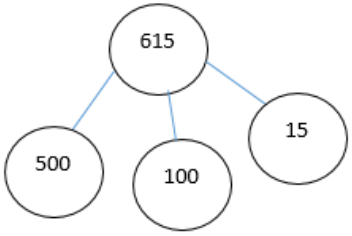


After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$$

Conceptual variation; different ways to ask children to solve 615 ÷ 5

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5\overline{)615}$$

615 ÷ 5 =

$\square$ = 615 ÷ 5

What is the calculation?
What is the answer?